STUDENT SUPPORT MATERIAL
Class XII
Mathematics

तत् त्वं पूषन् अपावृणु

केन्द्रीय विद्यालय संगठन

Session 2016-17

Kendriya Vidyalaya Sangathan
New Delhi

संतोष कुमार मल्ल, भा.प्र.से.
आयुक्त
Santosh Kumar Mall, I.A.S.
Commissioner

तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

केन्द्रीय विद्यालय संगठन
KENDRIYA VIDYALAYA SANGATHAN
18, संस्थागत क्षेत्र, शहीद जीत सिंह मार्ग, नई दिल्ली–110016
दूरभाष : 91–11–26512579, फैक्स : 91–11–26852680
18, Institutional Area, Shaheed Jeet Singh Marg, New Delhi-110016 (India)
Tel. : 91-11-26512579, Fax : 91-11-26852680
E-mail : commissioner@kvsedu.org, Website : www.kvsangathan.nic.in

A WORD TO MY DEAR STUDENTS

It gives me great pleasure in presenting the Students' Support Material to all KV students of class XII.

The material has been prepared keeping in mind your needs when you are preparing for final exams and wish to revise and practice questions or when you want to test your ability to complete the question paper in the time allotted or when you come across a question while studying that needs an immediate answer but going through the text book will take time or when you want to revise the complete concept or idea in just a minute or try your hand at a question from a previous CBSE Board exam paper or the Competitive exam to check your understanding of the chapter or unit you have just finished. This material will support you in any way you want to use it.

A team of dedicated and experienced teachers with expertise in their subjects has prepared this material after a lot of exercise. Care has been taken to include only those items that are relevant and are in addition to or in support of the text book. This material should not be taken as a substitute to the NCERT text book but it is designed to supplement it.

The Students' Support Material has all the important aspects required by you; a design of the question paper, syllabus, all the units/chapters or concepts in points, mind maps and information in tables for easy reference, sample test items from every chapter and question papers for practice along with previous years Board exam question papers.

I am sure that the Support Material will be used by both students and teachers and I am confident that the material will help you perform well in your exams.

Happy learning!

28/10

Santosh Kumar Mall
Commissioner, KVS

FOREWORD

The Students' Support Material is a product of an in-house academic exercise undertaken by our subject teachers under the supervision of subject expert at different levels to provide the students a comprehensive, yet concise, learning support tool for consolidation of your studies. It consists of lessons in capsule form, mind maps, concepts with flow charts, pictorial representation of chapters wherever possible, crossword puzzles, question bank of short and long answer type questions with previous years' CBSE question papers.

The material has been developed keeping in mind latest CBSE curriculum and question paper design. This material provides the students a valuable window on precise information and it covers all essential components that are required for effective revision of the subject.

In order to ensure uniformity in terms of content, design, standard and presentation of the material, it has been fine tuned at KVS Hqrs level.

I hope this material will prove to be a good tool for quick revision and will serve the purpose of enhancing students' confidence level to help them perform better. Planned study blended with hard work, good time management and sincerity will help the students reach the pinnacle of success.

Best of Luck.

U.N. Khaware
Additional Commissioner (Acad.)

STUDENT SUPPORT MATERIAL

ADVISORS

Shri Santosh Kumar Mall, IAS, Commissioner, KVS (HQ) New Delhi

Shri. U.N. Khaware, Addl. Commissioner (Academics), KVS (HQ)

CO-ORDINATION TEAM AT KVS (HQ)

Dr. V. Vijayalakshmi, Joint Commissioner (Acad), KVS (HQ)

Mr. P.V. Sai Ranga Rao, Deputy Commissioner (Acad), KVS (HQ)

Ms. Aprajita, AEO (Acad), KVS (HQ)

CONTENT TEAM

Mr. M.S. Chauhan, D.C., Patna Region (now at RO Chandigarh)

All Post Graduate Teachers of Patna Region

REVIEW TEAM

Mr. Ajay Gupta, PGT (Maths), KV CRPF, Jharondakalan, New Delhi

Mr. Nitin Maheshwari, PGT (Maths), KV Vikaspuri, New Delhi

Typing Type-setting & Designing

M/s Vijaylakshmi Printing Works Pvt. Ltd.

B-117, Sector-5, Noida-201301, Ph.: 0120-2421977, 2422312

E-mail: vpwpl.1972@gmail.com

INDEX

RELATIONS & FUNCTIONS

Gist of chapter:

(i) Domain, Co domain & Range of a relation

(ii) Types of relations

(iii) One-one , onto & inverse of a function

(iv) Composition of function

(v) Binary Operations

INVERSE TRIGONOMETRIC FUNCTIONS

Gist of chapter:

(i) Principal value

(ii) Properties of inverse trigonometric functions

(iii) Solution of Inverse T Equations

MATRICES & DETERMINANTS

Gist of chapter:

(i) Order, Addition, Multiplication and transpose of matrices

(ii) Cofactors &Adjoint of a matrix

(iii) Inverse of a matrix using elementary transformation

(iv) Inverse of a matrix & applications

(v) To find difference between $|A|$, $|adjA|$, $|kA|$, $|A.adjA|$

(vi) Properties of Determinants

CONTINUITY AND DIFFRENTIABILITY

Gist of chapter:

1. Limit of a function
2. Continuity
3. Differentiation
4. Logrithmic Differentiation
5 Parametric Differentiation
6. Differentiation Implicit Fucntions
7. Second order derivatives
8. Rolle's Theorem and Mean Value Theorem

APPLICATION OF DERIVATIVES

Gist of chapter:

1. Rate of change
2. Increasing& decreasing functions
3. Tangents & normals
4. Approximations
5. Maxima & Minima

INDEFINITE & DEFINITE INTEGRALS

Gist of chapter:

(i) Integration by substitution

(ii) Application of trigonometric function in integrals

(iii) Integration of some particular function

(iv) Integration using Partial Fraction

(v) Integration by Parts

(vi) Some Special Integrals

(vii) Miscellaneous Questions

(i) Definite Integrals based upon types of indefinite integrals

(ii) Definite integrals as a limit of sum

(iii) Properties of definite Integrals

(iv) Integration of modulus function

APPLICATIONS OF INTEGRATION

Gist of chapter:

(i) Area under Simple Curves

(ii) Area of the region enclosed between Parabola and line

(iii) Area of the region enclosed between Ellipse and line

(iv) Area of the region enclosed between Circle and line

(v) Area of the region enclosed between Circle and parabola

(vi) Area of the region enclosed between Two Circles

(vii) Area of the region enclosed between Two parabolas

(viii) Area of triangle when vertices are given

(ix) Area of triangle when sides are given

(x) Miscellaneous Questions

DIFFERENTIAL EQUATIONS

Gist of chapter:

(i) Order and degree of a differential equation

(ii) General and particular solutions of a differential equation

(iii) Formation of differential equation whose general solution is given

(iv) Solution of differential equation by the method of separation of variables

(vi) Homogeneous differential equation of first order and first degree

(vii) Solution of differential equation of the type $\frac{dy}{dx} + py = q$ where p and q are functions of x and solution of differential equation of the type $\frac{dx}{dy} + py = q$ where p and q are functions of y.

VECTOR ALGEBRA

Gist of chapter:

(i) Vector and scalars

(ii) Direction ratio and direction cosines

(iii) Unit vector

(iv) Position vector of a point and collinear vectors

(v) Scalar (Dot) product of two vectors

(vi) Projection of a vector

(vii) Vector (Cross) product of two vectors

(viii) Area of a triangle

(ix) Scalar Triple Product

THREE DIMENSIONAL GEOMETRY

Gist of chapter:

(i) Direction Ratios and Direction Cosines

(ii) Cartesian and Vector equation of a line in space & conversion of one into another form

(iii) Co-planer and skew lines

(iv) Shortest distance between two lines

(v) Cartesian and Vector equation of a plane in space & conversion of one into another form

(vi) Angle Between : (i) Two lines (ii) Two planes (iii) Line & plane

(vii) Distance of a point from a plane

(viii) Distance measures parallel to plane and parallel to line

(ix) Equation of a plane through the intersection of two planes

(x) Foot of perpendicular and image with respect to a line and plane

LINEAR PROGRAMMING

Gist of chapter:

(i) LPP and its Mathematical Formulation
(ii) Graphical method of solving LPP (bounded and unbounded solutions)
(iii) Diet Problem
(iv) Manufacturing Problem
(v) Allocation Problem
(vi) Miscellaneous Problems

PROBABILITY

Gist of chapter:

(i) Conditional Probability
(ii) Multiplication theorem on probability
(iii) Independent Events
(iv) Bayes' theorem, partition of sample space and Theorem of total probability
(v) Random variables & probability distribution
(vi) Mean& variance of random variables
(vii) Bernoulli's trials and Binomial Distribution

IMPORTANT TIPS FOR EXAMINATION

- Must write Question number correctly and boldly.
- Attempt all questions.
- Read the questions carefully and give precise answers(nothing more, nothing less).
- As far as possible attempt questions in same order.
- Must draw a line after every answer.
- If you have attempted a question more than once, cancel the answer which you feel is incorrect otherwise the first attempt will be checked which may be wrong.
- Make clear distinction between letters like n, h etc. which look alike.
- Make sure that questions involving figures are attempted with figures and answers on the same page, otherwise there may be mistakes committed while turning pages.
- Time management is important. If you are not able to attempt a particular question, just go to the next problem and do not stay stuck in that.
- Keep rough work separate from the main solution.
- Write answers giving enough space between two answers.
- Underline the final answer.
- Answers to word problems should be in sentence form.

SAMPLE QUESTION PAPER

BLUE PRINT
Mathematics
Class-XII (2016-17)

S. No.	Topic	VSA (1 Mark)	SA (2 Mark)	LA -I (4 Mark)	LA-II (6 Mark)	Marks	Total
1	Relations& Functions	2			1	8	10
2	Inverse Trigonometric Functions		1			2	
3	Matrices		1			2	13
4	Determinants	1		1	1	11	
5	Continuity & Differentiation		1	2		10	44
6	Application of differentiation		1	2		10	
7	Integration		1	1	1	12	
8	Application of Integration				1	6	
9	Differential Equation		1	1		6	
10	Vectors	1	1	1		7	17
11	Three Dimensional Geometry			1	1	10	
12	Linear Programming				1	6	6
13	Probability		1	2		10	10
	Total	4 (4)	8 (16)	11 (44)	6 (36)	**100**	

SAMPLE QUESTION PAPER
Mathematics (041)
Class-XII (2016-17)

Time allowed : **3** hours   Maximum Marks : **100**

General Instructions:

(i) **All** questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Question **13-23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.
(vi) Question **24-29** in **Section D** are long-answer-**II** type questions carrying **6** marks each.

Section-A

Questions 1 to 4 carry 1 mark each.

1. State the reason why the Relation $R=\left\{(a,b):a\le b^2\right\}$ on the set **R** of real numbers is not reflexive.

2. If A is a square matrix of order 3 and $|2A|=k|A|$, then find the value of k.

3. If $\vec{a}$ and $\vec{b}$ are two nonzero vectors such that $\left|\vec{a}\times\vec{b}\right|=\vec{a}\cdot\vec{b}$, then find the angle between $\vec{a}$ and $\vec{b}$.

4. If $*$ is a binary operation on the set **R** of real numbers defined by $a*b=a+b-2$, then find the identity element for the binary operation $*$.

Section-B

Questions 5 to 12 carry 2 marks each.

5. Simplify $\cot^{-1}\frac{1}{\sqrt{x^2-1}}$ for $x<-1$.

6. Prove that the diagonal elements of a skew symmetric matrix are all zeros.

7. If $y=\tan^{-1}\frac{5x}{1-6x^2}, -\frac{1}{\sqrt{6}}<x<\frac{1}{\sqrt{6}}$, then prove that $\frac{dy}{dx}=\frac{2}{1+4x^2}+\frac{3}{1+9x^2}$.

8. If x changes from 4 to 4.01, then find the approximate change in $\log_e x$.

9. Find $\int\left(\frac{1-x}{1+x^2}\right)^2 e^x dx$.

10. Obtain the differential equation of the family of circles passing through the points $(a,0)$ and $(-a,0)$.

11. If $\left|\vec{a}+\vec{b}\right|=60, \left|\vec{a}-\vec{b}\right|=40$ and $\left|\vec{a}\right|=22$, then find $\left|\vec{b}\right|$.

12. If $P(A)=\frac{2}{5}, P(B)=\frac{1}{3}, P(A\cap B)=\frac{1}{5}$, then find $P(\overline{A}/\overline{B})$.

Section-C

Questions 13 to 23 carry 4 marks each.

13. If $A=\begin{bmatrix}1 & -2\\ 2 & 1\end{bmatrix}$, then using A^{-1}, solve the following system of equations: $x-2y=-1, 2x+y=2$.

14. Discuss the differentiability of the function $f(x)=\begin{cases}2x-1, x<\frac{1}{2}\\ 3-6x, x\geq\frac{1}{2}\end{cases}$ at $x=\frac{1}{2}$.

OR

For what value of k is the following function continuous at $x=-\frac{\pi}{6}$?

$$f(x)=\begin{cases}\dfrac{\sqrt{3}\sin x+\cos x}{x+\frac{\pi}{6}}, x\neq-\frac{\pi}{6}\\ k, x=-\frac{\pi}{6}\end{cases}$$

15. If $x=a\sin pt, y=b\cos pt$, then show that $\left(a^2-x^2\right)y\frac{d^2y}{dx^2}+b^2=0$.

16. Find the equation of the normal to the curve $2y=x^2$, which passes through the point (2, 1).

OR

Separate the interval $\left[0,\frac{\pi}{2}\right]$ into subintervals in which the function $f(x)=\sin^4 x+\cos^4 x$ is strictly increasing or strictly decreasing.

17. A magazine seller has 500 subscribers and collects annual subscription charges of Rs.300 per subscriber. She proposes to increase the annual subscription charges and it is believed that for every increase of Re 1, one subscriber will discontinue. What increase will bring maximum income to her? Make appropriate assumptions in order to apply derivatives to reach the solution. **Write one important role of magazines in our lives.**

18. Find $\int\frac{\sin x}{\left(\cos^2 x+1\right)\left(\cos^2 x+4\right)}dx$.

19. Find the general solution of the differential equation $(1+\tan y)(dx-dy)+2xdy=0$.

OR

Solve the following differential equation: $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$.

20. Prove that $\vec{a}\cdot\{(\vec{b}+\vec{c})\times(\vec{a}+2\vec{b}+3\vec{c})\}=\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]$.

21. Find the values of 'a' so that the following lines are skew: $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-a}{4},\frac{x-4}{5}=\frac{y-1}{2}=z$.

22. A bag contains 4 green and 6 white balls. Two balls are drawn one by one without replacement. If the second ball drawn is white, what is the probability that the first ball drawn is also white?

23. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of diamond cards drawn. Also, find the mean and the variance of the distribution.

Section-D

Questions 24 to 29 carry 6 marks each.

24. Let $f:[0,\infty)\rightarrow \mathbf{R}$ be a function defined by $f(x)=9x^2+6x-5$. Prove that f is not invertible. Modify, only the codomain of f to make f invertible and then find its inverse.

OR

Let $*$ be a binary operation defined on $Q\times Q$ by $(a,b)*(c,d)=(ac,b+ad)$, where Q is the set of rational numbers. Determine, whether $*$ is commutative and associative. Find the identity element for $*$ and the invertible elements of $Q\times Q$.

25. Using properties of determinants, prove that $\begin{vmatrix} \frac{(a+b)^2}{c} & c & c \\ a & \frac{(b+c)^2}{a} & a \\ b & b & \frac{(c+a)^2}{b} \end{vmatrix}=2(a+b+c)^3$.

OR

If $p \neq 0, q \neq 0$ and $\begin{vmatrix} p & q & p\alpha+q \\ q & r & q\alpha+r \\ p\alpha+q & q\alpha+r & 0 \end{vmatrix} = 0,$ then, using properties of determinants, prove that at least one of the following statements is true: (a) p, q, r are in G. P., (b) α is a root of the equation $px^2+2qx+r=0.$

26. Using integration, find the area of the region bounded by the curves $y=\sqrt{5-x^2}$ and $y=|x-1|.$

27. Evaluate the following: $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x+\cos^4 x} dx.$

OR

Evaluate $\int_0^4 (x+e^{2x})dx$ as the limit of a sum.

28. Find the equation of the plane through the point (4, -3, 2) and perpendicular to the line of intersection of the planes x – y + 2z – 3 = 0 and 2x – y -3z = 0. Find the point of intersection of the line $\vec{r}=\hat{i}+2\hat{j}-\hat{k}+\lambda(\hat{i}+3\hat{j}-9\hat{k})$ and the plane obtained above.

29. In a mid-day meal programme, an NGO wants to provide vitamin rich diet to the students of an MCD school .The dietician of the NGO wishes to mix two types of food in such a way that vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food 1 contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C. Food 2 contains 1 unit per Kg of vitamin A and 2 units per kg of vitamin C. It costs Rs 50 per kg to purchase Food 1 and Rs 70 per kg to purchase Food 2. Formulate the problem as LPP and solve it graphically for the minimum cost of such a mixture?

MARKING SCHEME
Mathematics (041)
Class-XII (2016-17)

Section A

1. $\frac{1}{2} > \left(\frac{1}{2}\right)^3 \Rightarrow \left(\frac{1}{2}, \frac{1}{2}\right) \notin R$. Hence, R is not reflexive. [1]

2. $k = 2^3 = 8$ [1]

3. $\sin\theta = \cos\theta \Rightarrow \theta = 45^\circ$ [1]

4. $e \in R$ is the identity element for $*$ if $a * e = e * a = a \forall a \in R \Rightarrow e = 2$ [1]

Section B

5. Let $\sec^{-1} x = \theta$. Then $x = \sec\theta$ and for x < -1, $\frac{\pi}{2} < \theta < \pi$ [1/2]

Given expression = $\cot^{-1}(-\cot\theta)$ [1/2]

= $\cot^{-1}(\cot(\pi - \theta)) = \pi - \sec^{-1} x$ as $0 < \pi - \theta < \frac{\pi}{2}$ [1]

6. Let A be a skew-symmetric matrix. Then by definition $A' = -A$ [1/2]

$\Rightarrow$ *the* (i, j)*th element of* A' = *the* (i, j)*th element of* $(-A)$ [1/2]

$\Rightarrow$ *the* (j, i)*th element of* $A = -$*the* (i, j)*th element of* A [1/2]

For the diagonal elements $i = j \Rightarrow$ *the* (i, i)*th element of* $A = -$*the* (i, i)*th element of* A

$\Rightarrow$ *the* (i, i)*th element of* $A = 0$ Hence, the diagonal elements are all zero. [1/2]

7. $y = \tan^{-1} \frac{3x + 2x}{1 - 3x2x} = \tan^{-1} 3x + \tan^{-1} 2x$ [1]

$\frac{dy}{dx} = \frac{3}{1 + 9x^2} + \frac{2}{1 + 4x^2}$ [1]

8. Let $y = \log_e x, x = 4, \delta x = .01$ [1/2]

$\frac{dy}{dx} = \frac{1}{x}$ [1/2]

$dy = \left(\frac{dy}{dx}\right)_{x=4} \times \delta x = \frac{1}{400} = .0025$ [1]

9. Given integral $= \int \left(\frac{1}{1 + x^2} - \frac{2x}{\left(1 + x^2\right)^2} \right) e^x dx$ [1]

$= \frac{1}{1 + x^2} e^x + c$ as $\frac{d}{dx}(\frac{1}{1 + x^2}) = \frac{-2x}{\left(1 + x^2\right)^2}$ [1]

10. Let center of circle is (0, b) $x^2+(y-b)^2=a^2+b^2 or, x^2+y^2-2by=a^2$.........(1) [1]

$$2x+2y\frac{dy}{dx}-2b\frac{dy}{dx}=0 \Rightarrow 2b=\frac{2x+2y\frac{dy}{dx}}{\frac{dy}{dx}}\text{.........(2)}$$ [1/2]

Substituting in (1), $(x^2-y^2-a^2)\frac{dy}{dx}-2xy=0$ [1/2]

11. $|\vec{a}+\vec{b}|^2+|\vec{a}-\vec{b}|^2=2\left(|\vec{a}|^2+|\vec{b}|^2\right)$ [1]

$\Rightarrow|\vec{b}|^2=2116$ [1/2]

$\Rightarrow|\vec{b}|=46$ [1/2]

12. $P(\bar{A}/\bar{B})=\frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}$ [1/2]

$=\frac{1-P(A\cup B)}{1-P(B)}$ [1/2]

$=\frac{1-[P(A)+P(B)-P(A\cap B)]}{1-P(B)}$ [1/2]

$=\frac{7}{10}$ [1/2]

Section C

13. $|A|=5$ [1/2]

$adjA=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}$ [1+1/2]

$A^{-1}=\frac{adjA}{|A|}=\frac{1}{5}\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}$ [1/2]

Given system of equations is $AX=B$, where $X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}-1\\ 2\end{bmatrix}$ [1/2]

$X=A^{-1}B=\begin{bmatrix}\frac{3}{5}\\ \frac{4}{5}\end{bmatrix}$ [1/2]

$\Rightarrow x=\frac{3}{5}, y=\frac{4}{5}$ [1/2]

14. $Lf'(\frac{1}{2}) = \lim_{h\to 0^+} \frac{f(\frac{1}{2}-h) - f(\frac{1}{2})}{-h} = \lim_{h\to 0^+} \frac{2(\frac{1}{2}-h)-1-0}{-h} = 2$ [1+1/2]

$Rf'(\frac{1}{2}) = \lim_{h\to 0^+} \frac{f(\frac{1}{2}+h) - f(\frac{1}{2})}{h} = \lim_{h\to 0^+} \frac{3-6(\frac{1}{2}+h)-0}{h} = -6$ [1+1/2]

$Lf'(\frac{1}{2}) \neq Rf'(\frac{1}{2})$, f is not differentiable at $x = \frac{1}{2}$ [1]

OR

$\lim_{x\to -\frac{\pi}{6}} f(x) = \lim_{x\to -\frac{\pi}{6}} \frac{2\sin(x+\frac{\pi}{6})}{x+\frac{\pi}{6}}$ [2]

$= 2$ [1]

$f(-\frac{\pi}{6}) = k$ [1/2]

For the continuity of $f(x)$ at $x = -\frac{\pi}{6}, f(-\frac{\pi}{6}) = \lim_{x\to -\frac{\pi}{6}} f(x) \Rightarrow k = 2$ [1/2]

15. $\frac{dx}{dt} = ap\cos pt, \frac{dy}{dt} = -bp\sin pt$ [1]

$\frac{dy}{dx} = \frac{-bp\sin pt}{ap\cos pt} = -\frac{b}{a}\tan pt$ [1/2]

$\frac{d^2y}{dx^2} = \frac{-bp\sec^2 pt}{a}\frac{dt}{dx}$ [1]

$= \frac{-bp\sec^2 pt}{a} \times \frac{1}{pa\cos pt} = \frac{-b^2}{(a^2-x^2)y} \Rightarrow (a^2-x^2)y\frac{d^2y}{dx^2} + b^2 = 0$ [1+1/2]

16. Let the normal be at (x_1, y_1) to the curve $2y = x^2$. $\frac{dy}{dx} = x$ The slope of the normal at

$(x_1, y_1) = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}} = \frac{-1}{x_1}$ [1]

The equation of the normal is $y - y_1 = \frac{-1}{x_1}(x - x_1)$ [1/2]

The point (2, 1) satisfies it $1 - y_1 = \frac{-1}{x_1}(2 - x_1) \Rightarrow x_1 y_1 = 2$.....(1) [1/2]

Also, $2y_1 = x_1^2$......(2) [1/2]

Solving (1) and (2), we get $x_1 = 2^{\frac{2}{3}}, y_1 = 2^{\frac{1}{3}}$ [1/2]

The required equation of the normal is $x + 2^{\frac{2}{3}} y = 2 + 2^{\frac{2}{3}}$ [1]

OR

$f'(x) = 4\sin^3 x\cos x - 4\cos^3 x\sin x = -\sin 4x$ [1]

$f'(x) = 0 \Rightarrow x = \frac{\pi}{4}$ [1]

In the interval	Sign of f′(x)	Conclusion	Marks
$(0, \frac{\pi}{4})$	-ve as $0 < 4x < \pi$	f is strictly decreasing in $[0, \frac{\pi}{4}]$	[1]
$(\frac{\pi}{4}, \frac{\pi}{2})$	+ve as $\pi < 4x < 2\pi$	f is strictly increasing in $[\frac{\pi}{4}, \frac{\pi}{2}]$	[1]

17. Increase in subscription charges = Rs x, Decrease in the number of subscriber = x. Obviously, x is a whole number. [1/2]

Income is given by y = (500 – x)(300 + x). Let us assume for the time being

$0 \le x < 500, x \in R$ [1]

$\frac{dy}{dx} = 200 - 2x, \frac{dy}{dx} = 0 \Rightarrow x = 100$ [1/2]

$\frac{d^2y}{dx^2} = -2, \left(\frac{d^2y}{dx^2}\right)_{x=100} = -2 < 0$ [1/2]

y is maximum when x = 100, which is a whole number. Therefore, she must increase the subscription charges by Rs 100 to have maximum income. [1/2]

Magazines contribute, a great deal, to the development of our knowledge. Through valuable and subtle critical and commentary articles on culture, social civilization, new life style we learn a lot of interesting things. Through reading magazines, our mind and point of view are consolidated and enriched. [1]

18. $\cos x = t \Rightarrow -\sin x dx = dt$ The given integral $= -\int \frac{dt}{(t^2+1)(t^2+4)}$ [1]

Put $t^2 = y, \frac{-1}{(y+1)(y+4)} = \frac{A}{y+1} + \frac{B}{y+4}$ [1/2]

$-1 = (y+4)A + B(y+1), 0 = A + B, -1 = 4A + B \therefore A = \frac{-1}{3}, B = \frac{1}{3}$ [1]

The given integral $= -\frac{1}{3}\int \frac{dt}{(t^2+1)} + \frac{1}{3}\int \frac{dt}{(t^2+4)} = -\frac{1}{3}\tan^{-1} t + \frac{1}{6}\tan^{-1}\frac{t}{2} + c$ [1]

$= -\frac{1}{3}\tan^{-1}(\cos x) + \frac{1}{6}\tan^{-1}\frac{\cos x}{2} + c$ [1/2]

19. $\left(1+\tan y\right)dx=(1+\tan y-2x)dy \Rightarrow \dfrac{dx}{dy}+\dfrac{2}{1+\tan y}x=1$ [1]

$I.F.=e^{\int\frac{2dy}{1+\tan y}}=e^{\int\frac{(-\sin y+\cos y)+(\cos y+\sin y)}{\cos y+\sin y}dy}=e^{\log_e(\cos y+\sin y)+y}=(\cos y+\sin y)e^y$ [2]

$x(\cos y+\sin y)e^y=\int(\cos y+\sin y)e^y dy \Rightarrow x(\cos y+\sin y)e^y=e^y\sin y+c$ [1]

OR

We have $(1+e^{\frac{x}{y}})dx=\left(\dfrac{x}{y}-1\right)e^{\frac{x}{y}}dy \Rightarrow \dfrac{dx}{dy}=\dfrac{\left(\dfrac{x}{y}-1\right)e^{\frac{x}{y}}}{(1+e^{\frac{x}{y}})}=f(\dfrac{x}{y})$, hence homogeneous [1/2]

$x=vy, \dfrac{dx}{dy}=v+y\dfrac{dv}{dy}$ [1/2]

$\int\dfrac{1+e^v}{e^v+v}dv=-\int\dfrac{dy}{y}$ [1]

$\log_e\left|e^v+v\right|=-\log_e|y|+\log_e c$ [1]

$\Rightarrow \log_e\left|(e^v+v)y\right|=\log_e c$ [1/2]

$\Rightarrow (e^v+v)y=\pm c=A \Rightarrow (e^{\frac{x}{y}}+\dfrac{x}{y})y=A$, the general solution [1/2]

20. LHS $=\vec{a}\cdot(\vec{b}\times\vec{a}+2\vec{b}\times\vec{b}+3\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+2\vec{c}\times\vec{b}+3\vec{c}\times\vec{c})$ [1]

$=\vec{a}\cdot(\vec{b}\times\vec{a})+3\vec{a}\cdot(\vec{b}\times\vec{c})+\vec{a}\cdot(\vec{c}\times\vec{a})+2\vec{a}\cdot(\vec{c}\times\vec{b})$ as $\vec{b}\times\vec{b}=\vec{c}\times\vec{c}=\vec{0}$ [1]

$=3\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]+2\left[\vec{a}\ \ \vec{c}\ \ \vec{b}\right]$ [1]

$=3\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]-2\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]=\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]$ [1]

21. As $2:3:4\neq 5:2:1$, the lines are not parallel [1/2]

Any point on the first line is $(2\lambda+1, 3\lambda+2, 4\lambda+a)$ [1]

Any point on the second line is $(5\mu+4, 2\mu+1, \mu)$ [1]

Lines will be skew, if, apart from being non parallel, they do not intersect. There must not exist a pair of values of λ, μ, which satisfy the three equations simultaneously:

$2\lambda+1=5\mu+4, 3\lambda+2=2\mu+1, 4\lambda+a=\mu$Solving the first two equations, we get $\lambda=-1, \mu=-1$

[1]

These values will not satisfy the third equation if $a\neq 3$ [1/2]

22. Let E_1 = First ball drawn is white, E_2 = First ball drawn is green, A =Second ball drawn is white [1]

The required probability, by Bayes' Theorem, =

$$P(E_1/A)=\frac{P(E_1)\times P(A/E_1)}{P(E_1)\times P(A/E_1)+P(E_2)\times P(A/E_2)}$$ [1]

$$=\frac{\frac{6}{10}\times\frac{5}{9}}{\frac{6}{10}\times\frac{5}{9}+\frac{4}{10}\times\frac{6}{9}}=\frac{5}{9}$$ [2]

23. Let X denote the random variable. X=0, 1, 2 n = 2, p = ¼, q = ¾ [1/2]

x_i	0	1	2	Total	Marks
p_i	${}^2C_0\left(\frac{3}{4}\right)^2=\frac{9}{16}$	${}^2C_1\frac{1}{4}\left(\frac{3}{4}\right)=\frac{6}{16}$	${}^2C_2\left(\frac{1}{4}\right)^2=\frac{1}{16}$		[1+1/2]
x_ip_i	0	6/16	2/16	1/2	
$x_i^2p_i$	0	6/16	4/16	5/8	[1/2]

Mean = $\sum x_ip_i=\frac{1}{2}$ [1/2]

Variance = $\sum x_i^2p_i-\left(\sum x_ip_i\right)^2$ [1/2]

$=\frac{5}{8}-\frac{1}{4}=\frac{3}{8}$ [1/2]

Section D

24. $\forall x\in[0,\infty), y=9x^2+6x-5=(3x+1)^2-6\geq-5$..........(1) Range f = $[-5,\infty)\neq$ codomain f, hence, f is not onto and hence, not invertible [2]

Let us take the modified codomain f = $[-5,\infty)$ [1/2]

Let us now check whether f is one-one. Let $x_1,x_2\in[0,\infty)$ such that

$f(x_1)=f(x_2)\Rightarrow(3x_1+1)^2-6=(3x_2+1)^2-6\Rightarrow 3x_1+1=3x_2+1\Rightarrow x_1=x_2$ Hence, f is one-one.

Since, with the modified codomain = the Range f, f is both one-one and onto, hence invertible.

[1+1/2]

From (1) above, for any y $\in[-5,\infty), (3x+1)^2=y+6\Rightarrow x=\frac{\sqrt{y+6}-1}{3}$ [1]

$f^{-1}:[-5,\infty)\rightarrow[0,\infty), f^{-1}(y)=\frac{\sqrt{y+6}-1}{3}$ [1]

OR

Let $(a,b),(c,d)\in Q\times Q$. Then b + ad may not be equal to d + cb. We find that

$(1,2)*(2,3)=(2,5),(2,3)*(1,2)=(2,7)\neq(2,5)$ Hence, $*$ is not commutative. [1]

Let

$(a,b),(c,d),(e,f)\in Q\times Q,((a,b)*(c,d))*(e,f)=(ace,b+ad+acf)=(a,b)*((c,d))*(e,f))$

Hence, $*$ is associative. [1]

$(x,y)\in Q\times Q$ is the identity element for $*$ if

$(x,y)*(a,b)=(a,b)*(x,y)=(a,b)\forall(a,b)\in Q\times Q$ i.e.,

$(xa,y+xb)=(ax,b+ay)=(a,b)$ $i.e., xa=a, y+xb=b+ay=b$, (x, y) = (1, 0) satisfies these equations. Hence, (1, 0) is the identity element for $*$ [2]

$(c,d)\in Q\times Q$ is the inverse of $(a,b)\in Q\times Q$ if

$(c,d)*(a,b)=(a,b)*(c,d)=(1,0), i.e., (ac,b+ad)=(ca,d+cb)=(1,0)\Rightarrow c=\frac{1}{a}, d=\frac{-b}{a}$. The inverse of $(a,b)\in Q\times Q, a\neq 0$ is $(\frac{1}{a},\frac{-b}{a})$ [2]

25. LHS = $\frac{1}{abc}\begin{vmatrix}(a+b)^2 & c^2 & c^2\\ a^2 & (b+c)^2 & a^2\\ b^2 & b^2 & (c+a)^2\end{vmatrix}$ [1/2]

$$=\frac{1}{abc}\begin{vmatrix}(a+b+c)(a+b-c) & 0 & c^2\\ 0 & (b+c+a)(b+c-a) & a^2\\ (b+c+a)(b-c-a) & (b+c+a)(b-c-a) & (c+a)^2\end{vmatrix}(C_1\to C_1-C_3, C_2\to C_2-C_3)$$ [1]

$$=\frac{(a+b+c)^2}{abc}\begin{vmatrix}(a+b-c) & 0 & c^2\\ 0 & (b+c-a) & a^2\\ (b-c-a) & (b-c-a) & (c+a)^2\end{vmatrix}$$ [1/2]

$$=\frac{(a+b+c)^2}{abc}\begin{vmatrix}(a+b-c) & 0 & c^2\\ 0 & (b+c-a) & a^2\\ (-2a) & (-2c) & 2ca\end{vmatrix}(R_3\to R_3-(R_1+R_2))$$ [1]

$$=\frac{(a+b+c)^2}{abcca}\begin{vmatrix}(ac+bc-c^2) & 0 & c^2\\ 0 & (ba+ca-a^2) & a^2\\ (-2ac) & (-2ca) & 2ca\end{vmatrix}$$ [1/2]

$$=\frac{(a+b+c)^2}{abcca}\begin{vmatrix}(ac+bc) & c^2 & c^2\\ a^2 & (ba+ca) & a^2\\ 0 & 0 & 2ca\end{vmatrix}(C_1\to C_1+C_3, C_2\to C_2+C_3)$$ [1]

$$=\frac{(a+b+c)^2\,2c^2a^2}{abcca}\begin{vmatrix}(a+b) & c & c\\ a & (b+c) & a\\ 0 & 0 & 1\end{vmatrix}(C_1\to C_1+C_3, C_2\to C_2+C_3)$$ [1]

$$=\frac{(a+b+c)^2\,2}{b}\left(ab+ac+b^2+bc-ac\right)=2(a+b+c)^3$$ [1/2]

OR

Given equation $\Rightarrow \frac{1}{pq}\begin{vmatrix}pq & q^2 & pq\alpha+q^2\\ pq & pr & pq\alpha+pr\\ p\alpha+q & q\alpha+r & 0\end{vmatrix}=0$ [1]

$$\Rightarrow \frac{1}{pq}\begin{vmatrix}0 & q^2-pr & q^2-pr\\ pq & pr & pq\alpha+pr\\ p\alpha+q & q\alpha+r & 0\end{vmatrix}=0\ (R_1\to R_1-R_2)$$ [1]

$$\Rightarrow \frac{q^2-pr}{pq}\begin{vmatrix}0 & 1 & 1\\ pq & pr & pq\alpha+pr\\ p\alpha+q & q\alpha+r & 0\end{vmatrix}=0$$ [1]

$$\Rightarrow \frac{q^2-pr}{pq}p\begin{vmatrix}0 & 1 & 1\\ q & r & q\alpha+r\\ p\alpha+q & q\alpha+r & 0\end{vmatrix}=0$$ [1]

$$\Rightarrow \frac{q^2-pr}{q}\begin{vmatrix}0 & 0 & 1\\ q & -q\alpha & q\alpha+r\\ p\alpha+q & q\alpha+r & 0\end{vmatrix}=0(C_2\to C_2-C_3)$$ [1]

$\Rightarrow \frac{q^2-pr}{q}(q^2\alpha+rq+pq\alpha^2+q^2\alpha)=0\Rightarrow (q^2-pr)(2q\alpha+r+p\alpha^2)=0\Rightarrow q^2-pr=0$ (i.e., p, q, r are in GP) or $2q\alpha+r+p\alpha^2=0$ (i.e., α is a root of the equation $2qx+r+px^2=0$ [1]

26.

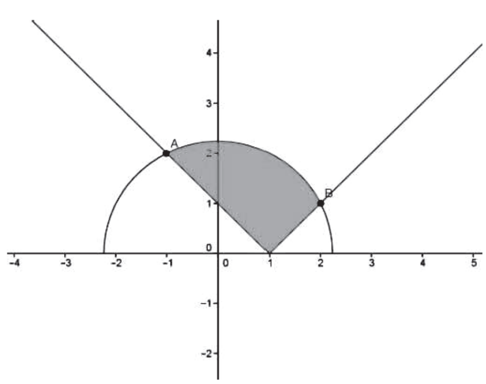


Figure [1 Marks]

Solving $y=\sqrt{5-x^2}$, $y=|x-1|$ we get $(x-1)^2=5-x^2\Rightarrow x^2-x-2=0\Rightarrow x=2,-1$ [1]

The required area = the shaded area = $\int_{-1}^{1}(\sqrt{5-x^2}-(1-x))dx+\int_{1}^{2}(\sqrt{5-x^2}-(x-1))dx$ [2]

$=\int_{-1}^{2}(\sqrt{5-x^2}dx-\int_{-1}^{1}(1-x)dx+\int_{1}^{2}(-(x-1))dx=\frac{1}{2}\left[x\sqrt{5-x^2}+5\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2}-\left[x-\frac{x^2}{2}\right]_{-1}^{1}-\left[\frac{x^2}{2}-x\right]_{1}^{2}$

[1+ ½]

$=\left[-\frac{1}{2}+\frac{5}{2}(\sin^{-1}\frac{2}{\sqrt{5}}+\sin^{-1}\frac{2}{\sqrt{5}})\right]$ sq units [1/2]

27. $I=\int_{0}^{\frac{\pi}{2}}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx=\int_{0}^{\frac{\pi}{2}}\frac{(\frac{\pi}{2}-x)\sin(\frac{\pi}{2}-x)\cos(\frac{\pi}{2}-x)}{\sin^4(\frac{\pi}{2}-x)+\cos^4(\frac{\pi}{2}-x)}dx$, $I=\int_{0}^{\frac{\pi}{2}}\frac{(\frac{\pi}{2}-x)\cos x\sin x}{\cos^4 x+\sin^4 x}dx$ [1]

$2I=\int_{0}^{\frac{\pi}{2}}\frac{(\frac{\pi}{2})\cos x\sin x}{\cos^4 x+\sin^4 x}dx$ [1/2],

$2I=(\frac{\pi}{2})[\int_{0}^{\frac{\pi}{4}}\frac{\cos x\sin x}{\cos^4 x+\sin^4 x}dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos x\sin x}{\cos^4 x+\sin^4 x}dx]=(\frac{\pi}{2})[\int_{0}^{\frac{\pi}{4}}\frac{\tan x\sec^2 x}{1+\tan^4 x}dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos ec^2 x\cot x}{\cot^4 x+1}dx]$ [2]

$2I=(\frac{\pi}{4})[\int_{0}^{1}\frac{1}{1+t^2}dt-\int_{1}^{0}\frac{1}{1+p^2}dp]$ substituting

$\tan^2 x=t,\cot^2 x=p\Rightarrow 2\tan x\sec^2 xdx=dt,-2\cot x\cos ec^2 xdx=dp$ [1]

$2I=(\frac{\pi}{4})[\tan^{-1}t]_0^1+(\frac{\pi}{4})[\tan^{-1}p]_0^1=\frac{\pi^2}{8}$ [1]

$I=\frac{\pi^2}{16}$ [1/2]

OR

$f(x)=x+e^{2x},\int_{0}^{4}f(x)dx=\lim_{n\to\infty,h\to 0}h\sum_{r=1}^{n}f(rh),nh=4$ [1]

$f(rh)=rh+e^{2rh},\sum_{r=1}^{n}f(rh)=h\sum_{1}^{n}r+\sum_{1}^{n}e^{2rh}$ [1]

$=h\frac{n(n+1)}{2}+e^{2h}\frac{e^{2nh}-1}{e^{2h}-1}$ [2]

$\int_{0}^{4}f(x)dx=\lim_{n\to\infty,h\to 0}[nh\frac{nh+h}{2}+e^{2h}\frac{e^8-1}{\frac{e^{2h}-1}{2h}}\times\frac{1}{2}]$ [1]

$=\lim_{h\to 0}[4\frac{4+h}{2}+e^{2h}\frac{e^8-1}{\frac{e^{2h}-1}{2h}}\times\frac{1}{2}]=8+\frac{e^8-1}{2}$ [1]

28. $\vec{n}=\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ 2 & -1 & -3\end{vmatrix}=5\hat{i}+7\hat{j}+\hat{k}$ [2]

The equation of the plane is $\vec{r}\cdot\vec{n}=(5\hat{i}+7\hat{j}+\hat{k})\cdot(4\hat{i}-3\hat{j}+2\hat{k}), i.e., \vec{r}\cdot(5\hat{i}+7\hat{j}+\hat{k})=1$ [1]

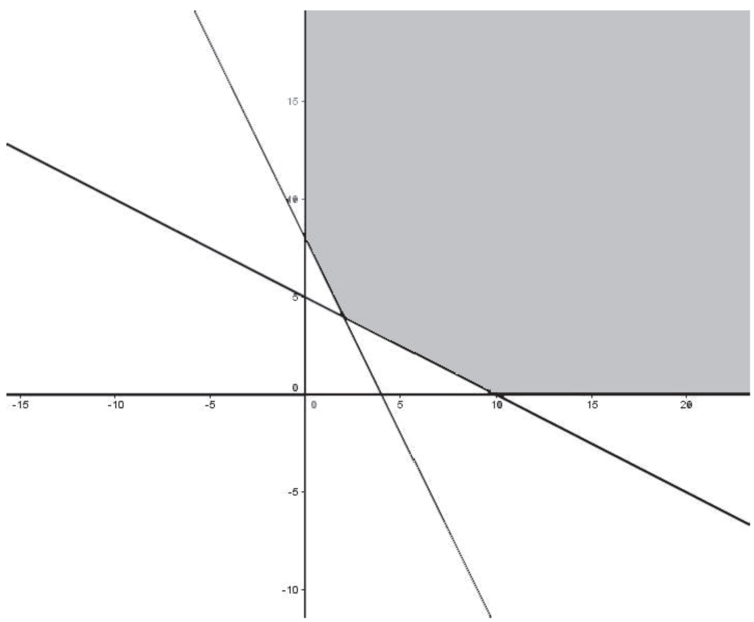
The position vector of any point on the given line is $(1+\lambda)\hat{i}+(2+3\lambda)\hat{j}+(-1-9\lambda)\hat{k}$ [1]

We have $(1+\lambda)5+(2+3\lambda)7+(-1-9\lambda)1=1$ [1]

$\lambda=-1$ [1/2]

The position vector of the required point is $-\hat{j}+8\hat{k}$ [1/2]

29. Let x kg of Food 1 be mixed with y kg of Food 2. Then to minimize the cost, C = 50x + 70y subject to the following constraints: $2x+y\geq 8, x+2y\geq 10, x\geq 0, y\geq 0$ [2]



Graph [2]

At	C	Marks
(0, 8)	Rs 560	[1]
(2,4)	Rs 380	
(10.0)	Rs 500	

In the half plane 50x + 70y < 380, there is no point common with the feasible region. Hence, the minimum cost is Rs 380. [1]

List of Important Formulas

RELATIONS & FUNCTIONS

1. (a) A relation in set A is a subset of A x A . We also write it as R = {(a, b) ∈ Ax A ;aRb}.

   (b) For relation R in set A, R^{-1} is inverse relation if $aR^{-1}b \Rightarrow b\ R\ a$.

2. A relation R in a set A is said to be reflexive , if (a, a) ∈ R, for every a ∈ A or we say aRa, for every a ∈ A.

3. A relation R in a set A is said to be symmetric , if (a, b) ∈ R ⇒ (b, a) ∈ R, for all a, b ∈ A. We can also say aRb, bRa for every a, b ∈ A.

4. A relation R in a set A is said to be transitive , if (a, b) ∈ R and (b, c) ∈ R ⇒ (a, c) ∈ R for every a, b, c ∈ A. We can also say aRb, bRc ⇒ aRc, for all a, b, c ∈ A.

5. A relation in a set A is said to be an equivalence relation if relation R reflexive, symmetric and transitive.

6. A function *f* is a rule from set A to set B which assigns to each elements of set A, a unique element of set B . Set A is called the domain of the function *f* set B is known as its co-domain. The set of values from set B which are actually taken by the function *f* is called the range of the function *f*.

   We denote it as $f: A \rightarrow B$, if x ∈ A then *f*(x) ∈ B.

7. A function whose domain and co – domain are the sets of real numbers is known as a real valued function, i.e $f: R \rightarrow R$.

8. One-one function : a function $f: A \rightarrow B$ is said to be one-one (or injective), if the images of distinct elements of A under the rule *f* are distinct in B, i.e for every a, b ∈ A, a ≠ b ⇒ *f* (a) ≠ *f*(b) or we also say that *f*(a) = *f*(b) ⇒ a = b .

9. Onto function $f: A \rightarrow B$ is said to be onto (or subjective), if every elements of B is image of some element of B is image of some element of A under the rule *f*, i.e for every b ∈ B, there exists an element a ∈ A such that *f* (a) = element a ∈ A such that *f* (a) = b.

   NOTE: - A function is onto if only if Range of function *f* = B.

10. One – one and onto function: A function $f : A \rightarrow B$ is said to be one-one and onto (bijective) if *f* is both one-one and onto.

11. Composition of function: Let $f: A \rightarrow B$ and : $B \rightarrow C$ be two given functions. Then the composition of function from A to C and is denoted by gof. We define gof as gof(x) = g{(*f*(x)} V x ∈ A . For working on element x first we apply *f* rule and whatever result is obtained in set B . we apply g rule on it to get the required result in set C.

12. A function $f: A \rightarrow B$ is said to be invertible , If there exists a function $g: B \rightarrow A$ such that $gof = I_A$ and $fog = I_B$. The function g is called the inverse of function *f* and is denoted by f^1.

13. Binary operation implies an operation involving the elements of a given set.

    A binary operation * on a set A is a function $* : A \times A \rightarrow A$ and * (a, b) is denoted by a * b.

14. A binary operation * on a set A is said to be communicative, if for all a , b ∈ A , a * b = b * a.

15. A binary operation * on a set A is said to be associative, If for all a, b , c ∈ A , (a * b) * c = a * (b * c).

16. Given a binary operation $* : A \times A \to A$, an element $e \in A$ is called identity element for the binary operation *, : a * e = e * a = a. for all $a \in A$.

17. Given a binary operation $* A \times A \to A$ with the identity element $e \in A$. An element $a \in A$ is said to be invertible with respect to the binary operation *, If there exists an element $b \in A$ such that

$a * b = e = b * a$, b is called inverse of a and denoted by a^{-1}

INVERSE TRIGONOMETRIC FUNCTIONS

FUNCTION	DOMAIN	RANGE (P. Value)
$\text{Sin}^{-1} x$	[-1,1]	[-π/2, π/2]
$\text{Cos}^{-1} x$	[-1,1]	[0, π]
$\tan^{-1} x$	*R*	(-π/2, π/2)

1. (i) $\text{Sin}^{-1}(\sin x) = x, [-\pi/2, \pi/2]$ (ii) $\text{Cos}^{-1}(\cos x) = x, [0, \pi]$

(iii) $\tan^{-1}(\tan x) = x, (-\pi/2, \pi/2)$

2. (i) $\text{Sin}^{-1}(-x) = -\text{Sin}^{-1}(x), x \in [-1, 1]$ (ii) $\text{cosec}^{-1}(-x) = -\text{cosec}^{-1} x,\ x \geq 1$

(iii) $\tan^{-1}(-x) = -\tan^{-1}(x), x \in \text{R}$ (iv) $\text{Cos}^{-1}(-x) = \pi - \cos^{-1}(x), x \in [-1, 1]$

(v) $\sec^{-1}(-x) = \pi - \sec^{-1}(x), |x| \geq 1$ (vi) $\cot^{-1}(-x) = \pi - \cot^{-1} x,\ x \in \text{R}$

3. (i) $\text{cosec}^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right), x \geq 1,\ x \leq -1$ (ii) $\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right), x \geq 1,\ x \leq -1$

(iii) $\cot^{-1}(\text{x}) = \tan^{-1}\left(\frac{1}{x}\right), x > 0$

4. (i) $\text{Sin}^{-1} x + \cos^{-1} x = \frac{\pi}{2}, x \in [-1, 1]$ (ii) $\text{cosec}^{-1}(x) + \sec^{-1}(x) = \frac{\pi}{2}, |x| \geq 1$

(iii) $\tan^{-1}(x) + \cot^{-1}(x) = \frac{\pi}{2},\ x \in \text{R}$

5. $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$, if $xy < 1$

$\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1} \frac{x+y}{1-xy}$, if $x,\ y > 0,\ xy > 1$

6. $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$, if $xy > -1$

7. $2 \tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2},\ -1 < x < 1$

$2 \tan^{-1} x = \cos^{-1} \frac{1-x^2}{1+x^2},\ -1 < x < 1$

$2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2},\ x \geq 0$

MATRICES & DETERMINANTS

Matrix is a rectangular array of numbers, real or complex kept inside braces [ ] or ( ) .

The numbers are called elements or members of matrix.

Order of a matrix: A matrix having "m" number of rows and "n" number of columns is of order "$m\times n$".

Types of matrices:

1. **Zero matrix:** Matrix having all elements 0 is called zero matrix and is denoted by "O"
2. **Square matrix:** Matrix having same number of rows and columns say "n" is called square matrix of order "n".
3. **Identity or unit matrix:** Square matrix $A = [a_{ij}]_m$ of order "m" is called identity or unit matrix if $a_{ij} = 0, i\ j$ & $a_{ij} = 1,\ i = j$. It is denoted by I e. g.

$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is a identity matrix of order 2 and $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is identity matrix of order 3.

Properties of Identity Matrices: (1) $A.I = I.A = A$ (2) $A.A^{-1} = A^{-1}.A = I$

Operations on matrices:

1. Addition of matrices: Only two matrices of same order can be added or subtracted by adding or subtracting corresponding elements of matrices.

   For matrices $A = [a_{ij}]_{m\times n}$ and $B = [b_{ij}]_{m\times n}$, $C = A + B$ with $C = [c_{ij}]_{m\times n}$ and $c_{ij} = a_{ij} + b_{ij}$ for all i, j.

   Properties (1) $A + B = B + A$ (2) $A + (B + C) = (A + B) + C$

   (3) $A + O = O + A = A$ (4) $A + (- A) = (- A) + A = 0$

2. Multiplication of matrix with scalar: For $A = [a_{ij}]_{m\times n}$ and scalar "k" then multiplication of A with k is denoted by "kA" and $kA = [k.a_{ij}]_{m\times n}$ e. g.

   For $A = \begin{bmatrix} -2 & 6 & 7 \\ 4 & 3 & 1 \end{bmatrix}$, $2A = \begin{bmatrix} -4 & 12 & 14 \\ 8 & 6 & 2 \end{bmatrix}$ and $-3\,A = \begin{bmatrix} 6 & -18 & -21 \\ -12 & -9 & -3 \end{bmatrix}$

3. Multiplication of matrices: Matrix A can be multiplied to matrix B if number of columns of A is same as number of rows of B. If $A = [a_{ij}]_{m\times n}$ and $B = [b_{jk}]_{n\times p}$ are two matrices then $A.B = [c_{ik}]_{m\times p}$ and $c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$

   Properties: (1) $A.B \neq B.A$ (2) $A.(B.C) = (A.B).C$

   (3) $A.(adjA) = (adjA).A = |A|.I$ (4) $A.I = I.A = A$ for square matrix A.

4. Transpose of a matrix: For matrix A = $[a_{ij}]_{m\times n}$, transpose of A is denoted by A^T or $A^/$ and $A^T=[a_{ji}]_{m\times n}$

e.g. $A = \begin{bmatrix} 1 & 2 \\ 9 & 4 \\ 2 & 3 \end{bmatrix}$, $A^T = \begin{bmatrix} 1 & 9 & 2 \\ 2 & 4 & 3 \end{bmatrix}$

Properties of transpose of matrix:

(1) $(A')' = A$, (2) $(A \pm B)^/ = A^/ \pm B^/$

(3) $(k A)^/ = k. A^/$ (4) $(A.B)^/ = B^/ A^/$

Symmetric matrix: Square matrix A = $[a_{ij}]$ is symmetric if $A^/$ = A i.e. $a_{ij} = a_{ji}$

Skew-symmetric matrix: Square matrix A = $[a_{ij}]$ is skew-symmetric if $A^/$ = - A i.e. $a_{ij} = -a_{ji}$ for all i, j Square matrix as sum of two matrices: For a square matrix A, A = P + Q where P is symmetric and Q is skew- symmetric.

Here $P = \frac{1}{2}(A + A^/)$ and $Q = \frac{1}{2}(A - A^/)$

Minor of an element: Minor of a_{ij} in matrix A is denoted by M_{ij} and is the determinant of order (n – 1) obtained by leaving the i th row and j th column of A

Co-Factor of an Element: Co-factor of a_{ij} in matrix A is denoted by C_{ij} and its value is given by $C_{ij} = (-1)^{i+j} M_{ij}$.

Important Results : (i) If A is square matrix of order n and A = k B then $|A| = k^n |B|$

(ii) If A is non singular matrix of order n and then $|adj\ A| = |A|^{n-1}$

Properties of determinants:

(i) The value of a determinant remains unchanged if its rows and columns are interchanged.

For $A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and $A^/ = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$ then A = $A^/$.

(ii) If any two rows or columns of a determinant are identical or proportional, value of determinant is 0. Also if all the values of a row/column are zero, value of it is zero.

$A = \begin{vmatrix} a_1 & a_1 & c_1 \\ a_2 & a_2 & c_2 \\ a_3 & a_3 & c_3 \end{vmatrix} = 0$ Or $A = \begin{vmatrix} 1 & 2 & 5 \\ 2 & 4 & -4 \\ 3 & 6 & 6 \end{vmatrix} = 0$; $B = \begin{vmatrix} 1 & 0 & 2 \\ -2 & 0 & 3 \\ 5 & 0 & 7 \end{vmatrix} = 0$

(iii) If any two rows or columns of a determinant are interchanged, value of determinant is multiplied by -1.

$$A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = - \begin{vmatrix} a_1 & c_1 & b_1 \\ a_2 & c_2 & b_2 \\ a_3 & c_3 & b_3 \end{vmatrix}$$

(iv) If each element of a row or column of a determinant is multiplied by a constant k, then value of the determinant gets multiplied by k.

$$A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ and } B = \begin{vmatrix} a_1 & b_1 & c_1 \\ k\,a_2 & k\,b_2 & k\,c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \text{ then } A = k.\ B$$

(v) If to any row or column of a determinant, a multiple of another row or column is added or subtracted, the value of the determinant remains unchanged. (At a time at least one row or column of the determinant should be kept unchanged)

$$A = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 + k\,a_2 & b_1 + k\,b_2 & c_1 + k\,c_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad R_1 \to R_1 + k\,R_2$$

$$\text{OR} = \begin{vmatrix} a_1 - b_1 & b_1 & c_1 \\ a_2 - b_2 & b_2 & c_2 \\ a_3 - b_3 & b_3 & c_3 \end{vmatrix} \quad C_1 \to C_1 - C_2$$

(vi) If each element of a row or column of a determinant is expressed as sum of two elements or more terms, then the determinant can be expressed as sum of two determinants keeping all other rows or columns same.

$$A = \begin{vmatrix} a_1 + x & b_1 + y & c_1 + z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} x & y & z \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}.$$

(vii) If A and B are two square matrices then $|A.B| = |A|.|B|$

Applications of determinants

Area of triangle: For a triangle having co-ordinates of vertices as $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3).

Area = $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

Cor. Points (x_1, y_1) (x_2, y_2) and (x_3, y_3).are collinear if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$.

Important results:

1. Square matrix A is singular then $|A| = 0$.
2. If A is a square matrix of order n, then $|kA| = k^n |A|$
3. If A is a non- singular matrix of order n, then $|Adj\ A| = |A|^{n-1}$
4. If A is a square matrix , then $|A^{-1}| = \frac{1}{|A|}$
5. If A = $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ then $A^n = \begin{bmatrix} 1 & na \\ 0 & 1 \end{bmatrix}$
6. If A = $\begin{bmatrix} a & 0 \\ 0 & a \end{bmatrix}$ then $A^n = \begin{bmatrix} a^n & 0 \\ 0 & a^n \end{bmatrix}$
7. If A is a skew-symmetric matrix of order n , then $(A^n)^T = (-1)^n A^n$

Adjoint of a square Matrix is transpose of matrix obtained by replacing each element of it by its cofactors and is denoted by adj(A)

Property of Adjoint: (Adj A). A = A. (Adj A) = $|A|$ I

Singular Matrix is the square matrix whose determinant is zero otherwise it is called non-singular matrix.

Note: Inverse of only nonsingular matrices exists.

Inverse of a square Matrix: A non-zero square matrix A of order n is called invertible matrix if there exists a square matrix B of order n such that AB = BA = I and B is called the inverse of A and vice versa. The inverse of A is denoted by A^{-1}.

Properties:(1) $A.A^{-1} = A^{-1}.A = I$ (2) $(AB)^{-1} = B^{-1}A^{-1}$ (3) $(A^{/})^{-1} = (A^{-1})^{/}$ (4) $Adj(A^{-1}) = (AdjA)^{-1}$

Solution of equations: For the equations

$a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

Equations in matrix form can be expressed as AX = B with

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, \; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

(1) If $|A| \neq 0$, then the system of equations is consistent and has a unique solution given by $X = A^{-1} B$

(2) If $|A| = 0$ and also (adj A) .B = 0, then the system of equations is consistent and has infinite many solutions.

(3) If $|A| = 0$ and also (adj A) .B $\neq$ 0, then the system of equations is inconsistent and has no solution.

CONTINUITY AND DIFFRENTIABILITY

Important formulas: (i) $\lim_{x\to a} \frac{x^n - a^n}{x - a} = n.a^{n-1}$ (ii) $\lim_{x\to 0} \frac{\tan x}{x} = 1$ (iii) $\lim_{x\to 0} \frac{\sin x}{x} = 1$

Continuity

A function y = f (x) is said to be continuous at x = a if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(x)$

Important formulas:

1. $\frac{d}{dx}(x^n) = n.\ x^{n-1}$
2. $\frac{d}{dx}\left(\frac{1}{x^n}\right) = \frac{-n}{x^{n+1}}$
3. $\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$
4. $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
5. $\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right) = -\frac{1}{2\ x^{3/2}}$
6. $\frac{d}{dx}(\alpha) = 0$
7. $\frac{d}{dx}(e^x) = e^x$
8. $\frac{d}{dx}(a^x) = a^x \log a$
9. $\frac{d}{dx}(\log x) = \frac{1}{x}$
10. $\frac{d}{dx}(\log_a x) = \frac{1}{x.\log a.}$
11. $\frac{d}{dx}(Sin\,x) = $ Cos x
12. $\frac{d}{dx}(Cos\,x) = $ - Sin x
13. $\frac{d}{dx}(\tan x) = \sec^2 x$
14. $\frac{d}{dx}(Co\sec x) = $ - Cosec x. Cot x

15. $\frac{d}{dx}(Sec\,x) =$ Sec x. tan x

16. $\frac{d}{dx}(Cot\,x) =$ - Cosec^2 x

17. $\frac{d}{dx}(Sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

18. $\frac{d}{dx}(Cos^{-1}x) = \frac{-1}{\sqrt{1-x^2}}$

19. $\frac{d}{dx}(\tan^{-1}x) = \frac{1}{1+x^2}$

20. $\frac{d}{dx}(Co\sec^{-1}x) = \frac{-1}{x\sqrt{x^2-1}}$

21. $\frac{d}{dx}(Sec^{-1}x) = \frac{1}{x\sqrt{x^2-1}}$

22. $\frac{d}{dx}(Cot^{-1}x) = \frac{-1}{1+x^2}$

Algebra of differentiation

1. $\frac{d}{dx}[k.f(x)] =$ k. $\frac{d}{dx}f(x)$

2. $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}f(x) \pm \frac{d}{dx}g(x)$

Derivative of product of functions (Product Rule):

$$\frac{d}{dx}[f(x).g(x)] = f(x).\frac{d}{dx}g(x) + g(x).\frac{d}{dx}f(x)$$

Derivative of quotient of functions (Quotient Rule):

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x).\frac{d}{dx}f(x) - f(x).\frac{d}{dx}g(x)}{[g(x)]^2}$$

Derivative of composite of functions (Chain Rule):

$$\frac{d}{dx}[fog(x)] = \frac{d}{dx}[f\{g(x)\}] = f'\{g(x)\}.g'(x)$$

ROLLE'S THEOREM

If a function $y = f(x)$ is such that

(i) $f(x)$ is continuous on [a, b]

(ii) $f(x)$ is derivable on (a, b)

(iii) $f(a) = f(b)$

then there exists at least one value c of x in (a, b) such that $f'(c) = 0$

LAGRANGE'S MEAN VALUE THEOREM

(i) $f(x)$ is continuous on [a, b]

(ii) $f(x)$ is derivable on (a, b)

then there exists at least one value c of x in *(a, b)* such that $f'(c) = \frac{f(b) - f(a)}{b-a}$

APPLICATION OF DERIVATIVES

TANGENT AND NORMAL

Slope of the tangent to the curve y = $f(x)$ at the point A (x_1, y_1) lying on it is $m = (dy/dx)_A$

Also slope of normal is given by m = -1 / (slope of tangent)

Equation of tangent and normal to the curve y = $f(x)$ at the point A (x_1, y_1) on the curve

$y - y_1 = m(x - x_1)$

INCREASING AND DECREASING FUNCTIONS

Increasing Function: Function $f(x)$ is increasing on (a, b) if

$x_1 < x_2 \Rightarrow f(x_1) < f(x_2) \;\; for \; all \; x_1, x_2 \in (a,b)$

Decreasing Function: Function $f(x)$ is decreasing on (a, b) if

$x_1 < x_2 \Rightarrow f(x_1) > f(x_2) \;\; for \; all \; x_1, x_2 \in (a,b)$

Steps to solve the problems:

(1) To show y = $f(x)$ is increasing or decreasing on (a, b), find $f'(x)$ & show that for increasing function $f'(x) > 0$ andfor decreasing function $f'(x) < 0$

(2) To find intervals in which y = $f(x)$ is increasing or decreasing in (a, b)

(i) Find $f'(x)$;

(ii) put $f'(x) = 0$ to find critical points like x_1 & x_2 *etc.*

(iii) With critical points construct intervals of increasing and decreasing e.g. $(a, x_1), (x_1, x_2), (x_2, b)$

(iv) For each interval check value of $f'(x)$ by substituting values from the intervals. If $f'(x)>0$ function is increasing on the interval and if $f'(x) < 0$ function is decreasing on the interval.

MAXIMUM AND MINIMUM

FIRST DERIVATIVE TEST

Minimum: Function $f(x)$ attains local minimum value at $x = a$ if there exist a neighborhood (a - h, a + h) such that $f'(a-h) < 0$ & $f'(a+h) > 0$

Maximum: Function $f(x)$ attains local maximum value at $x = a$ if there exist a neighborhood (a - h, a + h) such that $f'(a-h) > 0$ & $f'(a+h) < 0$

SECOND DERIVATIVE TEST

Minimum: Function f(x) attains local minimum value at x = a if $f''(a) > 0$

Maximum: Function f(x) attains local maximum value at x = a if $f''(a) < 0$

Steps to solve the problems: (i) Find $f'(x)$; (ii) put $f'(x) = 0$ to find points of maximum and minimum; (iii) find $f''(x)$; (iv) Use second derivative test to find points of max. and min.; (v) Substitute points of max./min. in f(x) to get local max/min values.

INDEFINITE & DEFINITE INTEGRALS

FORMULAS

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$;
2. $\int dx = x + c$;
3. $\int \sqrt{x} dx = \frac{2}{3} x^{\frac{3}{2}} + c$;
4. $\int \frac{dx}{x^2} = \frac{-1}{x} + c$;
5. $\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + c$;
6. $\int e^x dx = e^x + c$;
7. $\int a^x dx = \frac{a^x}{\log a} + c$;
8. $\int \frac{dx}{x} = \log|x| + c$
9. $\int \sin x dx = -\cos x + c$;
10. $\int \cos x dx = \sin x + c$;
11. $\int \sec^2 x dx = \tan x + c$;
12. $\int \sec x . \tan x dx = \sec x + c$;
13. $\int \cos ecx . \cot x dx = -\cos ecx + c$;
14. $\int \cos ec^2 x dx = -\cot x + c$
15. $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + c = -\cos^{-1} x + c$;
16. $\int \frac{dx}{x\sqrt{x^2-1}} = \sec^{-1} x + c = -\cos ec^{-1} x + c$
17. $\int \frac{dx}{1+x^2} = \tan^{-1} x + c = -\cot^{-1} x + c$;
18. $\int \sec x dx = \log|\sec x + \tan x| + c$;
19. $\int \cos ec\, x dx = \log|\cos ecx - \cot x| + c$;
20. $\int \tan x dx = \log|\sec x| + c$
21. $\int \cot x dx = \log|\sin x| + c$;
22. $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c$

23. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + c$;

24. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + c$;

25. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \sec^{-1} \frac{x}{a} + c$

26. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + c$;

27. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \log\left|x + \sqrt{x^2 - a^2}\right| + c$;

28. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \log\left|x + \sqrt{a^2 + x^2}\right| + c$;

29. $\int \sqrt{a^2 - x^2}\, dx = \frac{x\sqrt{a^2 - x^2}}{2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + c$

30. $\int \sqrt{x^2 - a^2}\, dx = \frac{x\sqrt{x^2 - a^2}}{2} - \frac{a^2}{2} \log\left|x + \sqrt{x^2 - a^2}\right| + c$;

31. $\int \sqrt{x^2 + a^2}\, dx = \frac{x\sqrt{x^2 + a^2}}{2} + \frac{a^2}{2} \log\left|x + \sqrt{x^2 + a^2}\right| + c$

INTEGRATION BY PARTS

$\int f(x).g(x)dx = f(x).G(x) - \int G(x).f'(x)dx$

G(x) is integration of g(x)

Here preference for first function in "BY PARTS" →

1 I Inverse Trigonometry Functions
2 L Logarithmic Functions
3 A Algebraic Functions
4 T Trigonometric Functions
5 E Exponential Functions

INTEGRATION BY PARTIAL FRACTION

1. $\frac{P(x)}{(x-\alpha)(x-\beta)} = \frac{A}{(x-\alpha)} + \frac{B}{(x-\beta)}$

2. $\frac{P(x)}{(x-\alpha)^2(x-\beta)} = \frac{A}{(x-\alpha)} + \frac{B}{(x-\alpha)^2} + \frac{C}{(x-\beta)}$

3. $\frac{P(x)}{(x^2+\alpha)(x-\beta)} = \frac{Ax+B}{(x^2+\alpha)} + \frac{C}{(x-\beta)}$

4. $\frac{P(x)}{(x^2+a)(x^2+b)} = \frac{A}{x^2+a} + \frac{B}{x^2+b}$

Fundamental theorem of calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$ Here $\int f(x)dx = F(x)$.

2. Integration as limit of sum:

$$\int_a^b f(x)dx = \lim_{h\to 0} h\,[f(a+h) + f(a+2h) + \text{------------} + f(a+(n-1)h)]$$

(i) $1 + 2 + 3 + \text{--------} + (n-1) = \frac{n(n-1)}{2}$;

$$1^2 + 2^2 + 3^2 + \text{--------} + (n-1)^2 = \frac{n(n-1)(2n-1)}{6}$$

$$1^3 + 2^3 + 3^3 + \text{------------} + (n-1)^3 = \left[\frac{n(n-1)}{2}\right]^2$$

(ii) $a + ar + ar^2 + \text{---------} + ar^n = a\,\frac{r^n - 1}{r - 1}$ iii) $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$

Properties of definite intrigation:

1. $\int_a^a f(x).dx = 0$ 2. $\int_a^b f(x).dx = -\int_b^a f(x).dx$ 3. $\int_0^a f(x).dx = \int_0^a f(a-x).dx$

4. $\int_a^b f(x).dx = \int_a^c f(x).dx + \int_c^b f(x).dx$ Here a < c < b

5. $$\int_{-a}^{a} f(x)\,dx = \begin{cases} 0 \ \ if \ \ f(-x) = -f(x) \\ 2\int_0^a f(x)\,dx \ \ if \ \ f(-x) = f(x) \end{cases}$$

6. $$\int_0^{2a} f(x)\,dx = \begin{cases} 0 \ \ if \ \ f(2a-x) = -f(x) \\ 2\int_0^a f(x)\,dx \ \ if \ \ f(2a-x) = f(x) \end{cases}$$

7. $\int_a^b f(x).dx = \int_a^b f(z)dz$

8. $\int_a^b f(x).dx = \int_a^b f(a+b-x).dx$

Points to Remember

- **Differential Equation :** Equation containing derivatives of a dependant variable with respect to an independent variable is called differential equation.
- **Order of a Differential Equation :** The order of a differential equation is defined to be the order of the highest order derivative occurring in the differential equation.
- **Degree of a Differential Equation :** Highest power of highest order derivative involved in the equation is called degree of differential equation where equation is a polynomial equation in differential coefficients.
- **Formation of a Differential Equation :** We differentiate the family of curves as many times as the number of arbitrary constant in the given family of curves. Now eliminate the arbitrary constants from these equations. After elimination the equation obtained is differential equation.
- **Solution of Differential Equation**

(i) **Variable Separable Method**

$$\frac{dy}{dx} = f(x, y)$$

We separate the variables and get

$$f(x)dx = g(y)dy$$

Then $\int f(x)\,dx = \int g(y)\,dy + c$ is the required solutions.

(ii) ***Homogenous Differential Equation :*** A differential equation of the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ where f(x, y) and g(x, y) are both homogeneous functions of the same degree in x and y i.e., of the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ is called a homogeneous differential equation.

For solving this type of equations we substitute y = vx and then $\frac{dy}{dx} = v + x\frac{dv}{dx}$. The equation can be solved by variable separable method.

(iii) ***Linear Differential Equation :*** An equation of the from $\frac{dy}{dx} + Py = Q$ where P and Q are constant or functions of x only is called a linear differential equation. For finding solution of this type of equations, we find integrating factor (I.F.) $= e^{\int P\,dx}$.

Solution is $y\,(\text{I.F.}) = \int Q.(\text{I.F.})\,dx + c$

VECTOR ALGEBRA

1. For point A (x, y, z), $\overrightarrow{OA} = x\hat{i} + y\hat{j} + z\hat{k}$

2. For points A (x_1, y_1, z_1) and B (x_2, y_2, z_2); $\overrightarrow{AB} = (x_2 - x_1)\,\hat{i} + (y_2 - y_1)\,\hat{j} + (z_2 - z_1)\,\hat{k}$

3. If $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$, then $|\vec{r}| = \sqrt{a^2 + b^2 + c^2}$

4. Two vectors $\vec{a}$ and $\vec{b}$ are collinear if $\vec{a} = k\,\vec{b}$. Points A,B and C are collinear if $\overrightarrow{AB} = k\,\overrightarrow{AC}$.

5. $\hat{a} = \frac{\vec{a}}{|\vec{a}|}$; $\vec{a} = |\vec{a}|\,.\,\vec{a}$

6. If P divides join of AB in m:n then

$\overrightarrow{OP} = \frac{m\overrightarrow{OB} + n\overrightarrow{OA}}{m+n}$(internally) ; $\overrightarrow{OP} = \frac{m\overrightarrow{OB} + n\overrightarrow{OA}}{m-n}$(externally); $\overrightarrow{OP} = \frac{\overrightarrow{OB} + \overrightarrow{OA}}{2}$(mid point)

7. For $\overrightarrow{OP} = x\hat{i} + y\hat{j} + z\hat{k}$ vector components are $x\hat{i}$, $y\hat{j}$ and $z\hat{k}$ & scalar components are x, y & z

8. For a vector $\vec{r} = a\hat{i} + b\hat{j} + c\hat{k}$; a, b, c are direction ratios.

For unit vector $\hat{r} = l\hat{i} + m\hat{j} + n\hat{k}$; l, m, n are direction cosines

$|\vec{r}| = \sqrt{a^2 + b^2 + c^2}$ and $l^2 + m^2 + n^2 = 1$

9. For direction ratios a, b and c, direction cosines can be calculated as

$l = \frac{a}{\sqrt{a^2+b^2+c^2}}$, $m = \frac{b}{\sqrt{a^2+b^2+c^2}}$, $n = \frac{c}{\sqrt{a^2+b^2+c^2}}$

SCALAR (DOT) PRODUCT:

1. $\vec{a}.\vec{b} = \vec{b}.\vec{a}$ Also $\vec{a}.(\vec{b} + \vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$

2. $\vec{a}^2 = \vec{a}.\vec{a} = |a|^2$; $\hat{i}\,.\,\hat{i} = \hat{j}\,.\hat{j} = \hat{k}.\hat{k} = 1$ For two perpendicular vectors $\vec{a}, \vec{b}$; $\vec{a}.\vec{b} = 0$;

$\hat{i}\,.\hat{j} = \hat{j}.\hat{k} = \hat{k}.\hat{i} = 0$

3. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a}.\vec{b}}{|\vec{b}|}$

4. For vectors $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ & $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$; $\vec{a}.\vec{b} = a_1a_2 + b_1b_2 + c_1c_2$

5. To find angle Θ between vectors $\cos\Theta = = \frac{\vec{a}.\vec{b}}{|\vec{a}||\vec{b}|} = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$,

   Condition for two vectors to be perpendicular $a_1a_2 + b_1b_2 + c_1c_2 = 0$

   Condition for two vectors to be parallel $\vec{a} = \lambda\vec{b}$ OR $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

6. $(\vec{a} \pm \vec{b})^2 = |\vec{a}|^2 + |\vec{b}|^2 \pm 2\,\vec{a}.\vec{b}$; $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$

   $(\vec{a} + \vec{b} + \vec{c})^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2.\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{c}.\vec{a}$

VECTOR(CROSS) PRODUCT

$\vec{a} \times \vec{b} = a.\, b \sin\theta\, \hat{n}$, with θ be the angle between the two vectors and $\hat{n}$ the unit vector perpendicular to both the vectors.

1. $\vec{a} \times \vec{b} \neq \vec{b} \times \vec{a}$; $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$ Also $\vec{a} \times (\vec{b}+\vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
2. $\vec{a} \times \vec{a} = 0$, $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$
3. Magnitude of cross-product $|\vec{a} \times \vec{b}| = |\vec{a}|\,.\,|\vec{b}|.\sin\Theta$
4. Unit vector perpendicular to the plane containing vectors $\vec{a}$ and $\vec{b}$, $\hat{n} = \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$
5. Vectors of magnitude 'k' perpendicular to $\vec{a}$ and $\vec{b}$ is given by $\pm k\frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$
6. For $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ & $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$; $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$
7. Geomatrically vector product represents the area of the parallelogram whose sides are represented by the two vectors i.e Area of parallelogram with consecutive sides represented by $\vec{a}$ and $\vec{b}$; Area $|\vec{a} \times \vec{b}|$
8. Area of triangle with sides $\vec{a}$ and $\vec{b}$; Area $= \frac{1}{2}|\vec{a} \times \vec{b}|$
9. Area of parallelogram with diagonals represented by $\vec{d}1, \vec{d}_2$; Area $= \frac{1}{2}|d_1 \times d_2|$
10. Lagrange's identity $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 . |\vec{b}|^2 - (\vec{a}.\vec{b})^2$

SCALAR TRIPLE PRODUCT:-

For vectors $\vec{a}$, $\vec{b}$ & c, scalar triple product is $(\vec{a} \times \vec{b}).\vec{c}$ OR $\vec{a}.(\vec{b} \times \vec{c})$ and is denoted by $[\vec{a}\ \vec{b}\ \vec{c}]$

Properties:

For a parallelepiped having its continuous edges represented by vectors $\vec{a}$, $\vec{b}$ & $\vec{c}$, volume of parallelepiped = $[\vec{a}\ \vec{b}\ \vec{c}]$

Three vectors $\vec{a}$, $\vec{b}$, & $\vec{c}$ are coplanar iff $[\vec{a}\ \vec{b}\ \vec{c}] = 0$

Points A, B, C and D are coplanar if $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}] = 0$

For vectors $\vec{a} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ & $\vec{b} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$ and $\vec{c} = a_3\hat{i} + b_3\hat{j} + c_3\hat{k}$

$$[\vec{a}\ \vec{b}\ \vec{c}] = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$[\vec{a}\ \vec{b}\ \vec{c}] = -[\vec{b}\ \vec{a}\ \vec{c}]$, $[\vec{a}\ \vec{b}\ \vec{a}] = 0$

THREE DIMENSIONAL GEOMETRY

STRAIGHT LINE

1. Line passing through point A (x_1, y_1, z_1) (P.V. $\vec{a}$) and parallel to $\vec{m} = a\,\hat{i} + b\hat{j} + c\hat{k}$

   Vector form $\vec{r} = \vec{a} + \lambda\vec{m}$

   Cartesian form: $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

2. A general point on the line is P $(a\lambda + x_1, b\lambda + y_1, c\lambda + z_1)$

3. Line passing through two points A (x_1, y_1, z_1) (P.V. $\vec{a}$) & B (x_2, y_2, z_2) (P.V. $\vec{b}$)

   Vector form $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

   Cartesian form: $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$

4. Angle between lines

   $\vec{r_1} = a_1 + \lambda m_1$, $\vec{r_2} = a_2 + \mu m_2$ **OR** $\frac{x - x_1}{a_1} = \frac{y - y_1}{b_1} = \frac{z - z_1}{c_1}$; $\frac{x - x_1}{a_2} = \frac{y - y_1}{b_2} = \frac{z - z_1}{c_2}$

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}} = \frac{\vec{m}_1.\vec{m}_2}{|\vec{m}_1|\,|\vec{m}\,|}$$

For **perpendicular** lines $\vec{m}_1.\vec{m}_2 = a_1\,a_2 + b_1\,b_2 + c_1\,c_2 = 0$

For **parallel** lines $\vec{m}_1 = \mu\,\vec{m}_2$; $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

5. **Shortest distance between two skew lines.**

For two lines $\vec{r}_1 = a_1 + \lambda\, m_1$; $\vec{r}_2 = a_2 + \mu\, m_2$

$$\text{S. D.} = \frac{\left[(\vec{a}_2 - \vec{a}_1).(\vec{m}_1 \times \vec{m}_2)\right]}{|\vec{m}_1 \times \vec{m}_2|} \quad \text{OR S.D.} = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{\sum (b_1c_2 - b_2c_1)^2}}$$

Condition for two lines to intersect $\left[(\vec{a}_2 - \vec{a}_1).(\vec{m}_1 \times \vec{m}_2)\right] = 0$

For parallel lines $\vec{r}_1 = a_1 + \lambda\, m$; $\vec{r}_2 = a_2 + \mu\, m$ S. D. $= \dfrac{|(\vec{a}_2 - \vec{a}_1) \times \vec{m}|}{|\vec{m}|}$

PLANE

Some Important Result/Concepts

- Equation of plane is ax + by + cx + d = 0 where a, b and c are direction ratios of normal to the plane.
- Equation of plane passing through a point (x_1, y_1, z_1) is a $(x - x_1) + b\,(y - y_1) + c\,(z - z_1) = 0$
- Equation of plane in intercept form is $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$, where a, b, c are interception the axis.
- Equation of plane in normal form $lx + my + nz = p$ where l, m, h are d.c. of normal to the plane P is length of perpendicular from origin to the plane.
- Equation of plane passing through three points. (x_1, y_1, z_1), (x_2, y_2, z_2) and (x_3, y_3, z_3).

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

- Equation of plane passing through two points (x_1, y_1, z_1), (x_2, y_2, z_3) and perpendicular to the plane, $a_1x + b_1y + c_1z + d_1 = 0$ or parallel to the line

$\dfrac{x - \alpha_1}{a_1} = \dfrac{y - \beta_1}{b_1} = \dfrac{z - \gamma_1}{c_1}$ is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \end{vmatrix} = 0$$

- Equation of plane passing through the point (x_1, y_1, z_1) and perpendicular to the plane $a_1x + b_1y + c_1z + d_1 = 0$, $a_2x + b_2y + c_2z + d_2 = 0$ or parallel to the lines.

$\frac{x-\alpha_1}{a_1} = \frac{y-\beta_1}{b_1} = \frac{z-\gamma_1}{c_1}$ and $\frac{x-\alpha_2}{a_2} = \frac{y-\beta_2}{b_2} = \frac{z-\gamma_2}{c_2}$ is

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

- Equation of plane containing the line $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and pas

through the point (x_2, y_2, z_2) is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \end{vmatrix} = 0$

- Condition for coplaner lines : $\frac{x-x_1}{a_1} = \frac{y-y_1}{b_1} = \frac{z-z_1}{c_1}$ and

$\frac{x-x_2}{a_2} = \frac{y-y_2}{b_2} = \frac{z-z_2}{c_2}$ are coplanar if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

and equation of common plane is $\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$

- Equation of plane passing through the intersection of two planes $a_1x + b_1y + c_1z = 0$, $a_2x + b_2y + c_2z = 0$ is $(a_2x + b_1y + c_1z) + \lambda\,(a_2x + b_2y + c_2z) = 0$

- Perpendicular distance from the point (x_1, y_1, z_1) to the plane

$ax + by + cz + d = 0$ is $\frac{a_1x + b_1y + c_1z + d}{\sqrt{a^2 + b^2 + c^2}}$

- Distance between two parallel planes $ax + by + cz + d_1 = 0$ and

$ax + by + cz + d_2 = 0$ is $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$

LINEAR PROGRAMMING Some important results/concepts

Solving linear programming problem using corner Point method. The method comprises of the following steps :

1. Find the feasible region of the linear programming problem and determine its corner points (vertices) either by inspection or by solving the two equations of the lilies intersecting at that point.
2. Evaluate the objective function Z = ax + by at each corner point. Let M and m respectively denote the largest and smallest values of these points.
3. (i) When the feasible region is bounded, M and m are the maximum and minimum values of Z.
   (ii) In case, the feasible region is unbounded, we have :
4. (a) M is the maximum value of Z, if the open half plane determined by $ax + by > M$ has no point in common with the feasible region. Otherwise, Z has no maximum value.
   (b) Similarly, m is the minimum value of Z, if the open half plane determined by $ax + by < m$ has no point in common with the feasible region. Otherwise, Z has no minimum value.

PROBABILITY Some important results/concepts

Conditional Probability: If E and F are two events associated with the same sample space of a random experiment, the conditional probability of the event E given that F has occurred, i.e. P(E|F) is given by $P(E) = \frac{P(E\cap F)}{P(F)}$ provided $P(F) \neq 0$

Multiplication rule of probability : $P(E\cap F) = P(E)P(F|E) = P(F)\,P(E|F)$

provided $P(E) \neq 0$ and $P(F) \neq 0$.

Independent Events : E and F are two events such that the probability of occurrence of one of them is not affected by occurrence of the other.

Let E and F be two events associated with the same random experiment, then E and F are said to be independent if $P(E\cap F) = P(E).P(F)$.

Bayes Theorem : If $E_1, E_2, \ldots\ldots\ldots E_n$ are n non empty events which constitute a partition of sample space S, i.e. $E_1, E_2, \ldots\ldots\ldots E_n$ are pairwise disjoint and $E_1 \cup E_2 \cup \ldots\ldots\ldots \cup E_n = S$ and A is any event of nonzero probability, then $\frac{P(E_i)P(A|E_i)}{\sum_{j=1}^{n} P(E_i)P(A|E_i)}$ for any i= 1,2,3,…., n

The probability distribution of a random variable X is the system of numbers

X :	X_1	X_2	....	X_n
P(X) :	P_1	P_2	....	P_n

Where, $\sum_{i=1}^{n} P_i = 1$, i = 1, 2, 3,….,

Binomial distribution: The probability of x successes P(X=x) is also denoted by P (x) and is given by $P(x) = {}^nC_x Q^{n-x} P^x_x$, x=0,1,…, n. (q=1−p)

RELATIONS AND FUNCTIONS

Domain, co domain, range of a relation. 'Types of relations.

Level-I

1. If A = {1,2,3,4,5}write the relation R such that a + b = 8, a.b $\in$ A write the domain, range & codomain
2. Define a relation R on the set N of natural number by

   R = {(x, y): y = x + 5 and x < 4, x, y $\in$N} Determine whether the relation R is reflexive, symmetric and transitive,
3. Show that the relation R in the set {1,2,3}given by {(1,2),(2,1)}is symmetric but neither reflexive nor transitive.

Level-II

1. Show that the relation R in the set N given by R = {(a. b) : a is divisible by b and a, b $\in$N} is reflexive and transitive but not symmetric.
2. Let R be the relation, in the set N given by R = {(a, b): a > b} show that the relation is neither reflexive nor symmetric but transitive.
3. Check whether the relation R is reflexive, symmetric and transitive

   R = {(x, y) : x-3y = 0) on A = {1,2 3,4.........14}.
4. Show that the relation R on the set *R* of real numbers defined as R = {(a. b) : a b^2} is neither reflexive nor symmetric nor transitive.

   CBSE (F) 2009
5. Let R be relation on *R* defined as $(a, b) \in R$ iff $1 + ab > 0$ $a, b \in R$

   (a) Show that R is reflexive

   (b) Show that R is symmetric

   (e) Show that R is not transitive.

Level-III

1. Show that the relation R on A = $\{x : x \in$ z. $0 < x < 12\}$ given by

   R={(a.b) : |a - b| is a multiple of 4}is an equivalence relation. Find the equivalence class of 2.

   CBSE (AI)2010
2. Let N be the set of natural numbers & R be the relation on N×N defined, by R={(a, b) R (c, d), iff *a*+*d*=*b*+*c*}show that R is an. equivalence relation.

   CBSE(AI)2010
3. If R_1 and R_2 are equivalence relations in a set A, show that $R_1 \cap R_2$ is also an equivalence relation.

4. Let R be a relation on the set A of ordered pairs of positive integers defined by (x.,y)R(u,v) if xu = yv. Show that R is an equivalence relation.
5. Show that the number of equivalence relations on the set {1, 2, 3}containing (1, 2) and (2,1) is two.
6. Let A= set of all triangles in a plane and R be defined by R={(T_1,T_2) : T_1, $T_2 \in$ A and $T_1 \sim T_2$} show that R is an equivalence relation Consider the right triangle as T, with side 3, 4, 5, T_2 with side 5. 12. 13. T_3 with side 6, 8, 10, which of the pairs are related.

One-one, onto & inverse of a function

Level-I

1. Show that function $f:R \to R$ defied by $f(x) = x^2$ is neither one-one nor onto.
2. Prove that the greatest integer function $f:R \to R$ given by $f(x) = [x]$ is neither one-one nor onto.
3. Show that the signum function $f:R \to R$ given by $f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$ is neither one-one nor onto.
4. If $f(x_x) = x^2 + \frac{1}{x^2}$ that find $f\left(\frac{1}{x}\right)$
5. Let $fix) = \frac{x-1}{x+1}$, $x \neq -1$, then find $f^{-1}(x)$

Level-II

1. Show that $f:N \to N$ given by $f(x) = \begin{cases} x+1, \text{ if } x \text{ is odd} \\ x-1, \text{ if } x \text{ is even} \end{cases}$ is both one-one and onto
CBSE(AI) 2012
2. If $f:R \to R$ defined as $f(x) = (3x+5)/2$. find $f^{-1}(x)$
3. Let A= {1,2,3}, B = {4,5,6,7}and let f = {(1,4), (2,5).(3,6)}be a function from A to B state whether f is one-one or not.
CBSE(AI) 2011
4. Show that the function $f:R \to R$ given by $f(x) = x^3$ is injective.
5. Show that the function $f:R \to R$ defined by $f(x) = 7 - 2x^5$, $x \in$ R is objective.
6. Show that the function for $A = R$ -{2/3}defined as $f(x) = (4x + 3)/(6x - 4)$ is one-one and onto, hence find f^{-1}
(CBSE DELHI 2013)
7. Write the number of all one-one functions on the set A= {a, b, c} to itself.

Level-III

1. Consider a function $f:R_+ \to (-5, \infty)$ defined $f(x) = 9x^3 + 6x - 5$. Show that f is invertible and $f^{-1}(y) = \frac{\sqrt{(y+6} - 1}{3}$ where $R_+ = (0, \infty)$

CBSE(F)2010

2. Consider $f:R_+ \to (4, \infty)$, given by $f(x) = x^2 + 4$. show that f invertible with the inverse f^{-1} of given by $f^{-1}(y) = \sqrt{y-4}$ where R is the set of all non-negative real numbers.

CBSE(AI)2013.

3. Show that the function $f:R \to \{x \in R : 1 < x < 1\}$ defined by $f(x) = x/(1 + |x|)$, $x \in R$ is one-one and onto function.

4. Let A = $R - \{3\}$ and B = $R - (1)$, consider the function $f:A \to B$ defined by $f(x) = (x-2)/(x-3)$, show that the f is one-one and onto. Hence find f^{-1}.

5. Show that $f:R \to R$ defined by $f(x) = x^3 + 4$ is one-one onto. Show that $f^{-1}(x) = (x-4)^{1/3}$

Composition of Functions

Level-I

1. Find $fof(x)$ if $f:R \to R$ is defined by $f(x) = x^2 - 3x + 2$

2. If $f(x) = |x|$, $g(x) = |5x-2|$, then find $fog(x)$.

3. If $f(x) = x + 7$, $g(x) = x - 7$, $x \in R$ then find $fog(7)$.

4. If $f(x) = (x - 1) /(x + 1)$, then show that

a) $f\left(\frac{1}{x}\right) = -f(x)$

b) $f\left(-\frac{1}{x}\right) = -\frac{1}{f(x)}$

Level-II

1. Let $f: R \to R$ be defined as $f(x) = 10x + 7$,

find the function $g:R \to R$ such that $gof = fog = 1_R$

CBSE(AI)2011

2. If $f: R \to R$ be defined as $f(x) = (3 - x^3)^{1/3}$ then find $fof(x)$

CBSE 2010

3. Let $f.g: R \to R$ be obtained by $f(x) = |x|$ and $g(x) = [x]$ where $[x]$ denotes the greatest integer function. Find $fog\,(5/2)$ and $gof\,(-\sqrt{2})$

4. If $y = f(x) = (3x + 4)/(5x - 3)$, then find $fof(x)$

5. If $f:R \to R$ & $g:R \to R$ be defined as $f(x) = x^2$ and $g(x) = 2x - 3$ find $fog\,(x)$

Binary Operation

Level-I

1. If the binary operation on the set of integers is defined by $a*b=a+3b^2$. then find the value of 2*4 CBSE 2009
2. Let *be a binary operation on N given by $a*b$ = LCM of a and b, find 5*7
3. Let * be a binary operation on the set of rational numbers defined as $a*b = ab/5$ write the identity of * if any
4. Let * be a binary operation on N given by $a*b$ = HCF of a, b where $a, b \in N$. write the value of 22*4 CBSE 2009

Level II

1. Let A = Q x Q and * be a binary operation on A defined by $(a, b)*(c.d) = (ac, ad+b)$ find
   (1) identify element of binary operation *
   (2) the invertible elements of binary operation *
2. If the operation * on $Q - \{1\}$defined by $a*b = a+b-ab$ for all $a, b \in Q - \{1\}$then
   (1) Is * commutative
   (2) is * associative
   (3) find the identity element
   (4) find the inverse of a for each a $\in Q$- {1}.
3. Examine which of the following is a binary operation.
   (1) $a*b = (a+b) / 2.\ a, b \in N$
   (2) $a*b = (a +b) / 2.a.b \in Q$ and for binary operation * check commutative and associative law.

Level III

1. Let A = N x N and * be the binary operation on A defined byu (a.b)*(c.d) = (a+c.b+d). show that * is commutative and associative find the identify element for * on A if any.
2. Define a binary operation * on the set (0, 1, 2, 3, 4, 5) as

   $$a*b = \begin{cases} a + b, & \text{if } a + b < 6 \\ a + b - 6, & \text{if } a + b \geq 6 \end{cases}$$

   Show that zero is the identity for this operation & each element a*0 of the set is invertible wth $6 - a$ being the inverse of a.
3. Consider the binary operation * : $R \times R \rightarrow R$ *and* $O{:}R \times R \rightarrow R$ defined as $a*b = |a - b|$ and $aob = a,\ \forall\ a,b \in R$. Show that * is communicative but not associative, o is associative but not commutative.

4. Let X be a non-empty set and * be a binary operation on *P(X)* (the power set of set *x* defined by A*B = A∪B ∀A, B ∈ *P(X)*. Prove that * is both commutative and associative on *P(X)*. Find the identity element with respect to * on *P(X)*

also show that φ ∈ *P(X)* is the only invertible element of *P(X)*

ANSWER

Domain. co domain, range of a relation. Types of relation.

Level:- I

1. R = {(3,5),(4,4), (5,3)}

   D = {3,4,5}

   Co-domain = A

   Range = {3,4,5}

2. Transitive but not reflexive

Level:- II

3. Transitive but not reflexive

Level:- III

1. {2,6,10}

6. T_1 is related to T_3

One-one, onto & inverse of a function

Level:- I

4. $f\left(\frac{1}{x}\right) = \frac{1}{x^2} + x^2$

5. $f^{-1}(x) = \frac{1+x}{1-x}$, $x \neq 1$

Level:- II

2 $\frac{2y-5}{3}$ 3. YES

6. $\vec{f} = \frac{4y+3}{6y-4}$ 7. 6

Level:- III

4. $\frac{3y-2}{y-1}$. $y \neq 1$

Composition of function

Level:- I

1. $x^4 - 6x^3 + 10x^2 - 3x$
2. $|5x - 2|$
3. 7

Level:- II

1. $\frac{y-7}{10}$ 2. x 3. 2, 1 4. x 5. $(2x-3)^2$

BINARY OPERATION

Level-I

1. 50 2. 35 3. 5 4. 2

Level-II

1. (1, 0), $\left(\frac{1}{a}, \frac{-b}{a}\right)$, 2. yes, 0 3. no, yes, commutative, not associative.

Level-III

1. not exist

4. X

INVERSE TRIGONOMETRIC FUCNTIONS

Principal Value

Level-I

Write the principal value of the following.

(i) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(ii) $\sin^{-1}\left(-\frac{1}{2}\right)$

(iii) $\tan^{-1}\left(-\sqrt{3}\right)$

(iv) $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

Level-II

1. Write the Principal value of :-

(i) $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

2. Prove that

(a) $\sin^{-1}\left(\sin\frac{4\pi}{3}\right) = -\frac{\pi}{3}$

(b) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = -\frac{\pi}{6}$

Properties of Inverse Trigonometric Functions

Level-I

1. Evaluate $\cot(\tan^{-1} a + \cot^{-1} a)$
2. Prove that $3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$
3. Find x if $\sec^{-1}\left(\sqrt{2}\right) + \mathrm{cosec}^{-1}x = \frac{\pi}{2}$

Level-II

1. Write the following in the simplest form:

$\tan^{-1}\left[\frac{\sqrt{1+x^2}}{x}\right], x \neq 0$

2. Prove that $\sin^{-1}\frac{8}{17}+\sin^{-1}\frac{3}{5}=\tan^{-1}\frac{77}{36}$

3. Prove that $\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}$

4. Prove that $2\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)+\tan^{-1}\left(\frac{1}{7}\right)=\tan^{-1}\left(\frac{31}{17}\right)$

5. Prove that $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)$

Level-III

1. Prove that $\cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]=\frac{x}{2}, x\in\left(0,\frac{\pi}{4}\right)$

2. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x}+\sqrt{1-x}}{\sqrt{1+x}-\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$

3. Solve $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$

4. Solve $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

5. Solve $\tan^{-1}\frac{x-1}{x-2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$

6. Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2}, x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

ANSWERS

Principal value

Level-I i. $\frac{\pi}{6}$ ii. $\frac{-\pi}{6}$

iii. $\frac{-\pi}{3}$ iv. $\frac{2\pi}{3}$

Level-II 1. π

Properties of Inverse Trigonometric Functions

Level-I 1. 0 3. $\sqrt{2}$

Level-II 1. $\frac{1}{2}\tan^{-1}x$

Level-III 3. $x = 1/2$ or $x = 1/3$ 4. $x = -8$ or $x = \frac{1}{4}$

5. $x = 1/\sqrt{2}$

Matrices and Determinants

Order, Addition, Multiplication and transpose of matrices :

Level-I

1. If a matrix has 5 elements what are the possible orders it can have?

2. Construct a 3 x 2 matrix whose elements are giben by $a_{ij} = \frac{1}{2}\left|i - 3_j\right|$

3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 0 & 2 \end{bmatrix}$, then find A - 2B

4. If $A = \begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & -1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$, write the order of AB and BA

Level-II

1. For the following matries A and B verify $(AB)^T = B^T A^T$

   Where $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, B = [-1 2 1]

2. Give example of matrices A and B such that AB $\neq$ 0. where 0 is a zero matrix and A, B are both non zero matrices.

3. If B is skew symmetric matrix write whether the matrix (ABA^T) is symmetric or skew symmetric,

4. If $A = \begin{bmatrix} 3 & 1 \\ 7 & 5 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, Find a and b so that $A^2 + aI = bA$

Level-III

1. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then find the value of $A^2 - 3A + 2I$

2. Express the matrix A as the sum of symmetric and a skew symmetric matrix,

   where : $A = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

3. If $A = \begin{bmatrix} a & b \\ 0 & 1 \end{bmatrix}$, prove that $A^n = \begin{bmatrix} a^n & \frac{b(a^n - 1)}{a - 1} \\ 0 & 1 \end{bmatrix}$, $n \in N$

Co-factors & adjoint of a matrix

Level-I

1. Find the cofator of a_{12} in $A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$

2. Find the adjoint of the matrix $A = \begin{bmatrix} 2 & -1 \\ 4 & 3 \end{bmatrix}$

Level-II

Verify $A(\text{adj}A) = (\text{adj}A)A = |A|\, I$ in questions 1 and 2.

(1) $A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

(2) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$

Inverse of a matrix & Applications

Level-I

1. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ write A^{-1} in term of A

2. If A is a square matrix satisfying $A^2 = I$, then what is the inverse of A?

3. For what value of k, the matrix $A = \begin{bmatrix} 2-k & 3 \\ -5 & 1 \end{bmatrix}$, is not invertible ?

Level-II

1. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, show that $A^2 - 5A - 14I = 0$, Hence find A^{-1} .

2. If A, B,C are three non zero square matrices of same order, find the condition on A such that $AB = AC \Rightarrow B = C$

Level-III

1. If $A = \begin{bmatrix} 2 & 3 & 1 \\ -3 & 2 & 1 \\ 5 & -4 & -2 \end{bmatrix}$, find A^{-1} and hence solve the following system of equations.

$2x - 3y + 5z = 11,\ 3x + 2y - 4z = -5,\ x + y - 2z = -3$

2. Using matrices, solve the following system of equation :

(a) x + 2y - 3z = – 4

2x + 3y +2z = 2

3x - 3y + 4z = 11

(b) 4x + 3y + 2z = – 4

4x + 3y + 2x = 60

x + 2y + 3x = 70

3. Find the product AB where $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

and use it to solve the equations

x - y = 3, 2x + 3y + 4z = 17, y + 2z = 7

4. Using matrices solve the following system of equations

$$\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4$$

$$\frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0$$

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2$$

5. Using elementary transformation find the inverse of the matrix $\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

Problems on |A|, adjA|, |KA|

Level-I

1. Evaluate $\begin{vmatrix} \cos 15^0 & \sin 15^0 \\ \sin 75^0 & \cos 75^0 \end{vmatrix}$

2. What is the value of $|3I|$, where I is the identity matrix of order 3?

3. If A is non-singular matrix of order 3 and $|A|$ =3, find $|A|$.

4. For what value of a, $\begin{bmatrix} -1 & 2a \\ 3 & 8 \end{bmatrix}$ is a singular matrix.

Level-II

1. If A is a square matrix of order 3 suh that $|\text{adj A}| = 64$, find $|A|$.

2. If A is non-singular matrix of order 3 and $|A| = 7$, then find $|\text{adj A}|$.

Level-III

1. If A = $\begin{vmatrix} a & 2 \\ 2 & a \end{vmatrix}$ and $|A|^3 = 125$, then find a.

2. A square matrix A, of order 3, has $|A| = 5$, find $|A \text{ . adj } A|$

Properties of Determinants

Level-I

1. Find positive value of x if $\begin{vmatrix} 16 & 3 \\ 5 & 2 \end{vmatrix} = \begin{vmatrix} 2x & 3 \\ 5 & x \end{vmatrix}$

2. Evaluate $\begin{vmatrix} a + ib & c + id \\ -c + id & a - ib \end{vmatrix}$

Level-II

Using properties of determinants prove the following

(1) $\begin{vmatrix} b + c & a & a \\ b & c + a & b \\ c & c & a + b \end{vmatrix} = 4abc$

(2) $\begin{vmatrix} 1 + a^2 - b^2 & 2ab & -2b \\ 2ab & 1 - a^2 + b^2 & 2a \\ 2b & -2a & 1-a^2 -b^2 \end{vmatrix} = (1+ a^2 + b^2)^3$

(3) $\begin{vmatrix} x & x^2 & 1+ px^3 \\ y & y^2 & 1+ py^3 \\ z & z^2 & 1+ pz^3 \end{vmatrix} = (1 + pxyz)\ (x-y)\ (y - z)\ (z - x)$

(4) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (a-b)\ (b-c)\ c-a)\ (a+b+c)$ [CBSE 2012]

Level-III

1. Using properties of determinants, solve the following for x:

(a) $$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

(b) $$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

(c) $$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

By using properties of determinants in questions 2 to 7 show that

2 $$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

3. $$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3-3abc$$

4. $$\begin{vmatrix} b^2c^2 & bc & b+c \\ c^2a^2 & ca & c+a \\ a^2b^2 & ab & a+b \end{vmatrix} = 0$$

5. $$\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (ab+bc+ca)^3$$

6. $$\begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$$

7. $\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$

ANSWERS

Matrices and deterices and determinants

Order, Addition, Multiplication and transpose of matrices :

LEVEL - 1 (1) $1 \times 5.\ 5 \times 1$ (2) $\begin{bmatrix} 1 & \frac{5}{2} \\ \frac{1}{2} & 2 \\ 0 & \frac{3}{2} \end{bmatrix}$

(3) $\begin{bmatrix} -3 & -4 & 1 \\ 1 & 1 & -1 \end{bmatrix}$ (4) $2 \times 2,\ 3 \times 3$

LEVEL - II (3) Skew Symmetric (4) a = 8, b = 8

LEVEL - III (1) $\begin{bmatrix} 1 & -1 & -1 \\ 3 & -3 & 4 \\ -3 & 2 & 0 \end{bmatrix}$

(2) $\begin{bmatrix} a & 1/2 & -5/2 \\ 1/2 & -2 & -2 \\ -5/2 & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & -5/2 & -3/2 \\ 5/2 & 0 & -3 \\ 3/2 & 3 & 0 \end{bmatrix}$

Co-factors & Adjoint of a matrix

LEVEL I (1) 46 (2) $\begin{bmatrix} 3 & 1 \\ -4 & 2 \end{bmatrix}$

Inverse of a Matrix & Applications

LEVEL - I (1) $A-1 = \text{A}$ (2) $A-1 = \text{A}$

(3) K = 17

LEVEL - II (1) $\begin{bmatrix} -2/14 & -5/14 \\ -4/14 & -3/14 \end{bmatrix}$

LEVEL - III (1) $x=1,\ y=2,\ z=3$ (2) $x=3,\ y=-2,\ z=1$

(3) AB = $6I$, $x = 2$, $y = -1$, $z = 4$

(4) $x = \frac{1}{2},\ y = -1,\ z = 1$

(5) $\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

Problems on |A|, adjA|, |KA|

LEVEL - I (1) $\frac{1}{2}$ (2) 27

(3) 24 (4) $4/3$

LEVEL - II (1) 8 (2) 49

LEVEL - III (1) a = 3 (2) 125

Properties of determinants

LEVEL - I (1) x = 4 (2) $a^2 + b^2 + c^2$

LEVEL - III 1(a) 4 1(b) 0, 0, 3a

1(c) $\frac{-a}{3}$

Continuity & Differentiability

Level-I

1. Examine the continuity of the function $f(x) = x^2 + 5$ at $x = -1$

2. Examine the continuity of the function $f(x) = \frac{1}{x+3}$. $x \in$ R.

Level-II

1. Given an example of a function which is continuous at $x = 1$, but not differentiable at $x = 1$

2. Find the relationship between "a" and "b" so that the function "f" defined by

$$f(x) = \begin{cases} ax+1 & \text{if } x \le 3 \\ bx+3 & \text{if } x > 3 \end{cases}$$ is continuous at $x = 3$

Level (III)

1. For what value of K, the function $f(x) = \begin{cases} \frac{1-\cos 4x}{8x^2}, & x \neq 0 \\ K, & x = 0 \end{cases}$ is continuous at x = 0?

2. Let $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x < \frac{\pi}{2} \\ a, & \text{if } x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi - 2x)^2}, & \text{if } x > \frac{\pi}{2} \end{cases}$

If $f(x)$ be continuous function at $x = \frac{\pi}{2}$, find a and b.

3. For what value of K. is the function

$$f(x) = \begin{cases} \frac{\sin x + x\cos x}{x}, & \text{when } x \neq 0 \\ K, & \text{when } x = 0 \end{cases}$$

continuous at $x = 0$?

4. Find the value of 'k' so that its function $f(x)$ is continous at the point at $x = \frac{\pi}{2}$

$$f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x}, & \text{if } x \neq \frac{\pi}{2} \\ 3 & \text{if } x = \frac{\pi}{2} \end{cases}$$

Differentiation
Level-I

1. Discuss the differentiability of the function

2. Differentiate $y = \tan^{-1} \dfrac{2x}{1 - x^2}$ w.r.t. 'x'

3. If $y = \cos x^3 . \sin^2 (x^5)$, find $\dfrac{dy}{dx}$

Level-II

1. Find $\dfrac{dy}{dx}$, if $y = \sin{-1} \dfrac{1 - x^2}{1 + x^2}, 0 < x < 1.$

2. Differentiate $y = \sec{-1} \dfrac{1}{2x^2 - 1}$, w.r.t. 'x' $0 < x <$.

3. Find $\dfrac{dy}{dx}$ if $y^x + x^y + x^x = a^b$

4. Find $\dfrac{dy}{dx}$ if $x = a(\cos t + \log \tan \dfrac{t}{2})$
   any $y = a \sin t$.

5. If $y = \sin^{-1} x$, Show that $(1 - x^2) \dfrac{d^2y}{dx^2} - x \dfrac{dy}{dx} = 0.$

6. Verify Rolle's theorem, for the function $f(x) = \sin x$, in $[0, \pi]$.

Level-III

1. Find $\dfrac{dy}{dx}$, if $y = \tan^{-1} \left[\dfrac{\sqrt{1 + x^2} - \sqrt{1 - x^2}}{\sqrt{1 + x^2} + \sqrt{1 - x^2}} \right]$

2. Find $\dfrac{dy}{dx}$, if $y = \cot^{-1} \left[\dfrac{\sqrt{1 + \sin x} - \sqrt{1 - \sin x}}{\sqrt{1 + \sin x} + \sqrt{1 - \sin x}} \right]$

3. If $y = \sin^{-1} \left(\dfrac{a + b \cos x}{b + a \cos x} \right)$

   Show that $\dfrac{dy}{dx} = \left(\dfrac{-\sqrt{b^2 - a^2}}{b + a \cos x} \right)$

4. If $\cos y = x \cos (a + y)$, with $\cos a \neq \pm 1$.

   Prove that $\dfrac{dy}{dx} = \dfrac{\cos^2 (a + y)}{\sin a}$

5. if $y = e^{a\cos^{-1}x}$, $-1 \le x \ge 1$

Show that $(1-x^2)\dfrac{d^2y}{dx^2} - x\dfrac{dy}{dx} - a^2y = 0$

6. For any positive constant 'a' and $\dfrac{dy}{dx}$ where

$y = {}_a t + \dfrac{1}{t}$ and $x = \left(t + \dfrac{1}{t}\right)^a$

7. Find $\dfrac{dy}{dx}$ if $y = (x\cos x)^x + (x\sin x)^{1/x}$

ANSWERS

Continuity

Level-I

1. Continuous
2. Not Continuous

Level-II

2. 3a – 2b = 2
3. Not Continuous

Level-III

1. 1
2. a = 1/2, b = 4
3. K = 2
4. K = 6

Differentiation

Level-I

1. Not differentiable
2. $\dfrac{2}{1+x^2}$
3. $10x^2 \sin x^5, \cos x^3 - 3x^2 \sin x^2, \sin^2 x^5$

Level-II

1. $\dfrac{-2}{1-x^2}$
2. $\dfrac{-2}{\sqrt{1-x^2}}$
3. $-\left[\dfrac{y^x, \log y + y, x^{y-1} + x^x(1+\log x)}{xy^{x-1} + x^y \log x}\right]$
4. $\tan t$
5. $C = \dfrac{\pi}{2}$

Level-III

1. $\dfrac{x}{\sqrt{1-x^4}}$ [Hint $x^2 = \cos 2\theta$]

2. $\dfrac{1}{2}$ [Hint $1 \pm \sin x = (\cos \dfrac{x}{2} \pm \sin x/2)^2$

6. $\dfrac{a^{\left(t+\frac{1}{t}\right)} \log a}{a\left(t+\dfrac{1}{t}\right)}$

7. $(x \cos x)^x [1 - x \tan x + \log (x \cos x)] + (x \sin x)^{1/x} \left[\dfrac{x \cot x + 1 - \log (x \sin x)}{x^2}\right]$

Application of derivative

Rate of Change

Level-I

1. A balloon, which always remains spherical, has variable diameter 3/2 ($2x$ +1). find the rate of change of its volume with respect to x.
2. The side of a square sheet is increasing at the rate of 4 cm per minute. At what rate is the area increasing when the side is 8 cm long?
3. The radius of a circle is increasing at the rate of 7.0 cm/sec. What is the rate of increase of its circumference ?

Level-II

1. Find the point on the curve $y^2 = 8x$ for which the abscissa and ordinate change at the same rate?
2. A man 2 metre high walks at a uniform speed of 6 km/h away from a lamp post 6 metre high. Find the rate at which the length of his shadow increases. Also find the rate at which the tip of the shadow is moving away from the lamp post.
3. The length of a rectangle is increasing at the rate of 3.5 cm/sec and its breadth is decreasing at the rate of 3 cm/sec. find the rate of change of the area of the rectangle when length is 12 cm and breadth is 8 cm.

Level-III

1. A particle moves along the curve 6 y = x^2 + 2. Find the points on the curve at which y – coordinate is changing 8 times as fast as the x – coordinate.
2. Water is leaking from a conical funnel at the rate of 5 cm^5/sec. If the radius of the base of the funnel is 10 cm and altitude is 20 cm. Find the rate at which water level is dropping when it is 5 cm from top.
3. From a cylindrical drum containing petrol and kept vertical, the petrol is leaking at the rate of 10 ml/sec. If the radius of the drum is 10 cm and height 50 cm. find the rate at which the level of the petrol is changing when petrol level is 20 cm from bottom.

Increasing & decreasing functions

Level-I

1. Show that $f(x) = x^3 - 6x^2 + 18x + 5$ is an increasing function for all x $\in$ R.
2. Show that the function $x^2 - x + 1$ is neither increasing nor decreasing on (0.1).
3. Find the intervals in which the function $f(x)$ = sin x – cos x, $0 \leq x \leq 2\pi$ is increasing or decreasing.
4. Find the intervals in which the function

   $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is strictly increasing.

Level-II

1. Indicate the interval in which the function $f(x) = \cos x$, $0 \leq x \leq 2\pi$ is decreasing.
2. Show that the function $f(x) = \dfrac{\sin x}{x}$ is strictly decreasing on $(0, \pi/2)$
3. Find the intervals in which the function $f(x) = \dfrac{\log x}{x}$ increasing or decreasing.

Level-III

1. Find the interval of monotonocity of the function $f(x) = 2x^2 - \log x, x \neq 0$
2. Prove that the function $y = \frac{4 \sin \theta}{2 + \cos \theta} - \theta$ is an increasing function of in θ in $(0. \frac{\pi}{2})$

Tangents & Normals

Level-I

1. Find the equations of the normals to the curve $3x^2 - y^2 = 8$ which are parallel to the line $x + 3y = 4$
2. Find the point on the curve $y = x^2$ where the slope of the tangent is equal to the x - coordinate of the point.
3. At what points on the circle $x + y^3 - 4y + 1 = 0$, the tangent is parallel to x axis ?

Level-II

1. Find the equation of the normal to the curve $ay^2 = x^3$ at the point $(\mathrm{am}^2, \mathrm{am}^3)$.
2. For the curve $y = 2x^2 + 3x + 18$. Find all the points at which the tangent passes through the origin.
3. Find the equation of the normals to the curve $y = x^3 + 2x + 6$ which are parallel to the line $x + 14y + 4 = 0$.
4. Show that the equation of tangent at (x_1, y_1) to the parabola $y^2 = 4ax$ is $yy_1 = 2a(x + x_1)$.

Level-III

1. Find the equation of the tangent line to the curve $y = \sqrt{5x - 3} - 2$ which is parallel to the line
   $4x - 2y + 3 = 0$.
2. Show that the curve $x^2 - + y^2 - 2x = 0$ and $x^2 + y^2 - 2y = 0$ cut orthogonally at the point (0, 0)
3. Find the equation of tangent to the curve

   $x = \sin 3t, y = \cos 2t$ at $t = \pi/4$
4. Find the condition for the curves $\frac{x^2}{a^2} - \frac{y^2}{b^2} =$ and $xy = c^2$ to intersect orthogonally.

Approximations

Level-I

1. Evaluate $\sqrt{25.3}$ using differentiation
2. Use differentials to approximate the cube root of 66.
3. Evaluate $\sqrt{0.082}$ using differentiation
4. Evaluate $\sqrt{49.5}$ using differentiation

Level-II

1. If the radius of a sphere is measured as 9 cm with an error of 0.03 cm. then find the approximate error in calculating its surface area.

Maxima and Minima

Level-I

1. Find the maximum and minimum value of the function $f(x) = 3 - 2 \sin x$.
2. Show that the function $f(x) = x^3 + x^2 + x + 1$ has neither a maximum value nor a minimum value.
3. Find two positive numbers whose sum is 24 and whose product is maximum.

Level-II

1. Prove that the area of a right-angled triangle of given hypotenuse is maximum when the triangle is isosceles.
2. A piece of wire 28 (units) long is cut into two pieces. One piece is bent into the shape of a circle and other into the shape of a square. How should the wire be cut so that the combined area of the two figures is as small as possible.
3. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the windows to admit maximum light through the whole opening.

Level-III

1. Find the area of the greatest isosceles triangle that can be inscribed in a given ellipse having its vertex coincident with one extremity of major axis.
2. An open box with a square base is to be made out of a given quantity of card board of area c^2 square units. Show that the maximum volume of the box is $\frac{c^3}{6\sqrt{3}}$ cubic units.
3. Show that the rectangle of maximum perimeter which can be inscribed in a circle of radius r is square of side $r\sqrt{2}$.
4. A windows is in the shape of a rectangle surmounted by an equilateral triangle. If the perimeter of the window is 12 m. find the dimensions of the rectangle that will produce the largest area of the window.

Application of Derivatives

Answer

RATE OF CHANGE

Level-I

1. $(27/8)\ \pi\ (2x + 1)^2$
2. 64 sqcm/min
3. $1.4\ \pi$ cm/s

Level-II

1. (2.4)
2. 3 km/h
3. Decreasing at the rate of 8 sqem/s

Level-III

1. (4.11) & (–4, –31/3)
2. (4/45π) cm/s
3. 1/10π cm/s

INCREASING & DECREASING

Level-I

3. Increasing in $\left[0.\frac{3\pi}{4}\right]\cup\left[\frac{7\pi}{4},\ 2\pi\right]$, Decreasing in $\left[\frac{3\pi}{4},\frac{7\pi}{4}\right]$
4. Strictly increasing : (2, ∞), (–1, 0); strictly decreasing (– ∞, –1), (0, 2)

Level-II

1. [0.π]
2. Increasing in (1.e) and Decreasing in (e.∞)

Level-III

1. Decreasing in (0.1/2) & Increasing in (1/2.∞)

TANGENTS & NORMAL

Level-I

1. (2, 2) & –2, –2)
2. (0, 0)
3. (1, 0) & 1, 4)

Level-II

1. $2x - 3my - am^2 (2 + 3m^2) = 0$
2. (3, 45) & (–3, 27)
3. $x + 14y - 254 = 0$: $x + 14y + 86 = 0$

Level-III

1. $80x - 40y - 103 = 0$
2. b = ± a

APPROXIMATION

Level-I

1. 5.03
2. 4.041
3. 0.29867
4. 7.035

Level-II

1. 2.16 π SQCM

MAXIMA & MINIMA

Level-I

1. MAX = 5 & MIN = 1
2. 12, 12

Level-II

1. 112/(π + 4) UNITS, 28p/(π + 4) UNITS
2. L = 20/(π + 4) M, B = 10/(π + 4)M

Level-III

1. 3 3 / 4 ab
2. L = $4(6 + \sqrt{3})$ / 11m B = $(25 - 6\sqrt{3})$ / 11m

INDEFINITE INTEGERALS

Integration by Substitution

Level-I

(1) $\int \frac{\sec^2(\log x)\,dx}{x}$ (2) $\int \frac{e^{m\tan^{-1}x}\,dx}{1+x^2}$ (3) $\int \frac{e^{\sin^{-1}x}\,dx}{\sqrt{1-x^2}}$

Level-II

(1) $\int \frac{dx}{\sqrt{x}+x}$ (2) $\int \frac{dx}{x\sqrt{x^6-1}}$ (3) $\int \frac{dx}{e^x-1}$

Application of trigonometric functions in integrals

Level-I

(1) $\int \sin^3 x\,dx$ (2) $\sin 3x \cos 4x\,dx$ (3) $\cos x \cos 2x \cos 3x\,dx$

Level-II

(1) $\int \frac{1}{\cos(x-a)\cos(x-b)}$ (2) $\int \sec^4 x \tan x\,dx$ (3) $\int \frac{\sin 4x}{\sin x}\,dx$

Level-III

(1) $\int \cos^5 x\,dx$ (2) $\int \sin^2 x \cos^2 x\,dx$ (3) $\int \frac{\sin x}{\sin(x-a}\,dx$

Integration of some particular functions

Level-I

(1) $\int \frac{dx}{\sqrt{4x^2-9}}$ (2) $\int \frac{dx}{x^2+2x+10}$ (3) $\int \frac{\sec^2 x}{\sqrt{\tan^2 x+4}}$

Level-II

(1) $\int \frac{x\,dx}{x^4+x^2+1}$ (2) $\int \frac{\cos x}{\sin^2 x+4\sin x+5}\,dx$ (3) $\int \frac{dx}{\sqrt{7-6x-x^2}}$

Level-III

(1) $\int \frac{2x}{\sqrt{1-x^2-x^4}}$ (2) $\int \frac{x+3}{(x-1)\,(x-2)\,(x-3)}\,dx$ (3) $\int \frac{(x+2)}{\sqrt{x^2+5x+6}}\,dx$

(4) $\int \sqrt{\frac{1-x}{1+x}}\,dx$ (5) $\int \frac{(6x+7)}{(x-5)\,(x-4)}\,dx$

Integration using partial fraction

Level-I

(1) $\int \frac{(2x+1)}{(x+1)(x-1)}dx$ (2) $\int \frac{x^2}{(x-1)(x-2)(x-3)}dx$ (3) $\int \frac{(3x-2)}{(x+1)^2(x+3)}dx$

Level-II

(1) $\int \frac{x^2+2x+8}{(x-1)(x-2)}dx$ (2) $\int \frac{x^2+x+1)}{x^2(x+2)}dx$ (3) $\int \frac{(x^2+1)dx}{(x-1)^2(x+3)}$

Level-III

(1) $\int \frac{dx}{(x+2)(x^2+4)}$ (2) $\int \frac{dx}{\sin x+\sin 2x}$ (3) $\int \frac{(1-x^2)dx}{x(1-2x)}$

Integration by parts

Level-I

(1) $\int x \text{ xin } x\, dx$ (2) $\int \log x\, dx$ (3) $\int e^x (\tan x + \log \sec x)\, dx$

Level-II

(1) $\int \sin^{-1} x\, dx$ (2) $\int x^2 \sin^{-1} x\, dx$ (3) $\int \frac{x \sin^{-1} x\, dx}{\sqrt{1-x^2}}$

Level-III

(1) $\int \cos (\log x)\, dx$ (2) $\int \frac{e^x(1+x)}{(2+x)^2}$ (3) $\int \frac{\log x}{(1+\log x)^2}dx$

(4) $\int \frac{(2+\sin x)e^x dx}{1+\cos 2x}$ (5) $\int e^{2x} \cos 3x\, dx$

Some special integrals

Level-I

(1) $\int \sqrt{4+x^2}\, dx$ (2) $\int \sqrt{1-4x^2}\, dx$

Level-II

(1) $\int \sqrt{x^2+4x+6}\, dx$ (2) $\int \sqrt{1-4x-x^2}\, dx$

Level-III

(1) $\int (x+1)\sqrt{1-x-x^2}\, dx$ (2) $\int (x-5)\sqrt{x^2+x}\, dx$

Miscellaneous questions

LEVEL-I

(1) $\int \frac{dx}{x - x^3}$ (2) $\int \cos 6x \sqrt{1 + \sin 6x}\, dx$

LEVEL-II

(1) $\int \frac{1}{2 - 3\cos 2x} dx$ (2) $\int \frac{1}{3 + \sin 2x} dx$ (3) $\int \frac{dx}{4\sin^2 x + 5\cos^2 x}$

(4) $\int \frac{dx}{1 + 3\sin^2 x + 8\cos^2 x}$ (5) $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$ (6) $\int \frac{\sec x}{5\sec x + 4\tan x} dx$

LEVEL-III

(1) $\int \frac{3\sin x + 2\cos x}{3\cos x + 2\sin x} dx$ (2) $\int \frac{dx}{1 - \tan x}$ (3) $\int \frac{x^4}{x^4 - 1} dx$

(4) $\int \frac{x^2 + 1}{x^4 + x^2 + 1} dx$ (5) $\int \frac{x^2 - 1}{x^4 + 1} dx$ (6) $\int \sqrt{\tan x}\, dx$

(7) $\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$

Answers

Integration by Substitution

LEVEL-I (1) $\tan(\log x) + c$ (2) $\frac{1}{m} e^{\tan^{-1} x} + c$ (3) $e^{\sin^{-1} x} + c$

LEVEL-II (1) $2 \log\left|1 + \sqrt{x}\right| + c$ (2) $\frac{1}{3}\sec^{-1} x^3 + c$ (3) $\log_e\left|1 - e^x\right|$

LEVEL-III (1) $2\sqrt{\tan x} + c$ (2) $-\tan^{-1}(\cos x) + c$ (3) $\frac{\tan^2 x}{2} + \log_e\left|\tan x\right| + c$

Application of trigonometric functions in integrals

LEVEL-I (1) $\frac{-3}{4}\cos x + \frac{1}{12}\cos 3x + c$ (2) $-\frac{1}{14}\cos 7x + \frac{1}{2}\cos x + c$

(3) $\frac{x}{4} + \frac{\sin 6x}{4} + \frac{\sin 4x}{16} + \frac{\sin 2x}{8} + c$

LEVEL-II (1) $\frac{1}{\sin(a - b)} \log_e\left|\frac{\cos(x - a)}{\cos(x - b)}\right| + c$ (2) $\frac{\tan^2 x}{2} + \frac{\tan^4 x}{4} + c$

(3) $\frac{2}{3}\sin 3x + 2\sin x + c$

LEVEL-III (1) $\sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + c$ (2) $\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

(3) $\sin a \log_e \left|\sin(x - a)\right| + x \cos a + c$

Integration of some particular functions

LEVEL-I (1) $\frac{1}{2}\log_e\left|x+\frac{1}{2}\sqrt{4x^2-9}\right|+c$ (2) $\frac{1}{3}\tan^{-1}\left(\frac{x+1}{3}\right)+c$ (3) $\log_e\left|\tan x+\sqrt{\tan^2 x+4}\right|+c$

LEVEL-II (1) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)+c$ (2) $\tan^{-1}(\sin x+2)$ (3) $\sin^{-1}\left(\frac{2x-1}{5}\right)+c$

LEVEL-III (1) $\sin^{-1}\left(\frac{2x^2-1}{5}\right)+c$ (2) $\frac{1}{2}\log\left|x^2-2x-5\right|+\frac{2}{\sqrt{6}}\log\left|\frac{x-1-\sqrt{6}}{x-1+\sqrt{6}}\right|+c$

(3) $\sqrt{x^2+5x+6}-\frac{1}{2}\log\left|\left(x+\frac{5}{2}\right)+\sqrt{x^2+5x+6}\right|+c$ (4) $\sin^{-1}x+\sqrt{1-x^2}+c$

(5) $6\sqrt{x^2-9x+20}+34\log\left|\left(\frac{2x-9}{2}\right)+\sqrt{x^2-9x+20}\right|+c$

Integration using partial fraction

LEVEL-I (1) $\frac{1}{3}\log(x+1)+\frac{5}{3}\log(x-2)+c$

(2) $\frac{1}{2}\log(x-1)-2\log(x-2)+\frac{3}{2}\log(x-3)+c$ (3) $\frac{11}{4}\log\left(\frac{x+1}{x+3}\right)+\frac{5}{2(x+1)}+c$

LEVEL-II (1) $x-11\log(x-1)+16\log(x-2)+c$

(2) $\frac{1}{4}\log x-\frac{1}{2x}+\frac{3}{4}\log(x+2)+c$ (3) $\frac{3}{8}\log(x-1)-\frac{1}{2(x-1)}+\frac{5}{8}(x+3)+c$

LEVEL-III (1) $\log(x+2)-\frac{1}{2}\log(x^2+4)+\tan^{-1}x$

(2) $\frac{\log(1-\cos x)}{6}+\frac{\log(1+\cos x)}{2}-\frac{2}{3}\log(1+2\cos x)+c$

(3) $\frac{1}{3}\log(1+x)-\frac{1}{6}\log(1-x+x^2)+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right)+c$ (4) $\frac{x}{2}+\log x-\frac{3}{4}\log(1-2x)+c$

Integration by parts (Answer)

LEVEL-I (1) $-x\cos x+\sin x+c$ (2) $x\log x-x+c$ (3) $e^x\log\sec x+c$

LEVEL-II (1) $x\sin^{-1}x+\sqrt{1-x^2}+c$ (2) $\frac{x^3}{3}\sin^{-1}x+\frac{(x^2+2)\sqrt{1-x^2}}{9}+c$

(3) $-\sqrt{1-x^2}\sin^{-1}x+x+c$ (4) $2x\tan^{-1}x-\log(1+x^2)+c$

(5) $\frac{1}{2}[\sec x.\tan x+\log(\sec x+\tan x)]+c$

LEVEL-III (1) $\frac{x}{2}[\cos(\log x)+\sin(\log x)]+c$ (2) $\frac{e^x}{2+x}+c$ (3) $\frac{x}{1+\log x}+c$

(4) $e^x\tan x+c$ (5) $\frac{e^{2x}}{13}(3\sin 3x+2\cos 3x)+c$

Some special integrals

LEVEL-I (1) $\frac{x\sqrt{4+x^2}}{2}+2\log\left|x+\sqrt{4+x^2}\right|+c$ (2) $\frac{x\sqrt{1-4x^2}}{2}+\frac{1}{4}\sin^{-1}2x+c$

LEVEL-II (1) $\frac{(x+2)\sqrt{x^2+4x+6}}{2}+\log\left|(x+2)+\sqrt{x^2+4x+6}\right|+c$

(2) $\frac{(x+2)\sqrt{1-4x-x^2}}{2}+\frac{5}{2}\sin^{-1}\left(\frac{x+2}{\sqrt{5}}\right)+c$

LEVEL-III (1) $-\frac{1}{3}(1-x-x^2)^{3/2}+\frac{1}{8}(2x-1)\sqrt{1-x-x^2}+\frac{5}{16}\sin^{-1}\left(\frac{2x+1}{\sqrt{5}}\right)+c$

(2) $\frac{1}{3}(x^2+x)^{3/2}-\frac{11}{8}(2x+1)\sqrt{x^2+x}+\frac{11}{16}\log\left|(2x+1)+2\sqrt{x^2+x}\right|+c$

Miscellaneous Questions

LEVEL-I (1) $\frac{1}{2}\log\left|\frac{x^2}{1-x^2}\right| + c$ (2) $\frac{1}{9}(1+\sin 6x)^{3/2} + c$

LEVEL-II (1) $\frac{1}{2\sqrt{5}}\log\left|\frac{\sqrt{5}\tan x - 1}{\sqrt{5}\tan x + 1}\right| + c$ (2) $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{3\tan x + 1}{2\sqrt{2}}\right) + c$

(3) $\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{5}}\right) + c$ (4) $\frac{1}{6}\tan^{-1}\left(\frac{2\tan x}{3}\right) + c$ (5) $\tan^{-1}(\tan^2 x) + c$

LEVEL-III (1) $-\frac{12}{13}x - \frac{5}{13}\log|3\cos x + 2\sin x| + c$ (2) $\frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + c$

(3) $x + \frac{1}{4}\log\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\tan^{-1}x + c$ (4) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{3}x}\right) + c$

(5) $\frac{1}{2\sqrt{2}}\log\left|\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}x+1}\right| + c$ (6) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x}}\right) + \frac{1}{2\sqrt{2}}\left|\frac{\tan x - \sqrt{2\tan x} + 1}{\tan x + \sqrt{2\tan x} + 1}\right| + c$

(7) $\sqrt{2}\tan^{-1}\left(\frac{\tan x - 1}{\sqrt{2\tan x}}\right) + c$

DEFINITE INTEGRATION

Definite integrals as a limit of sum.

Level - I

1. Evaluate $\int_0^2 (x+2)\,dx$ as the limit of a sum.

2. Evaluate $\int_0^4 (4-x)\,dx$ as the limit of a sum.

Level - II

1. Evaluate $\int_1^2 (3x^2-1)\,dx$ as the limit of a sum.

2. Evaluate $\int_0^3 (x^2+1)\,dx$ as the limit of a sum.

Level - III

1. Evaluate $\int_1^2 (x^2+x+2)\,dx$ as the limit of a sum.

2. Evaluate $\int_2^4 (x^2-3x+2)\,dx$ as the limit of a sum.

Properties of definite integrals

Definit Integrals based upon types fo Indefinite integrals.

Level - I

Evaluate :

1. $\int_0^1 \frac{2x+3}{5x^2+1}\,dx$ 2. $\int_0^{\pi/2} \sqrt{\sin x}\cos^5 x\,dx$ (3) $\int_0^2 x\sqrt{x+2}\,dx$

Level - II

Evaluate :

1. $\int_1^2 \frac{5x^2}{x^2+4x+3}\,dx$ 2. $\int_1^2 \left(\frac{1}{x}-\frac{1}{2x^2}\right)e^{2x}\,dx$

Level - III

1. Evaluate : $\int_0^{\pi/4} \frac{\sin x+\cos x}{9+16\sin 2x}\,dx$ 2. Evaluate : $\int_{\pi/6}^{\pi/3} \frac{\sin x+\cos x}{\sqrt{\sin 2x}}\,dx$

Properties of definite integrals

Evaluate using properties of definite Integral.

Level - I

1. $\int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x} dx$

2. $\int_0^{\pi/2} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}} dx$

3. $\int_1^3 \frac{\sqrt{4-x}}{\sqrt{x}+\sqrt{4-x}} dx$

4. $\int_{-\pi/2}^{\pi/2} \sin^3 x \, dx$

Level - II

1. $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$

2. $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$

3. $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$

4. $\int_0^{\pi/2} \log(\tan x).dx$

Level - III

1. $\int_0^{\pi} \log(1+\cos x)dx$

2. $\int_0^{\pi/4} \log(1+\tan x)dx$

3. $\int_0^{\pi} \frac{xdx}{1+\sin x}$

Level - III

Integration of modulus functions

Evaluate using properties of definite Integral.

1. $\int_2^5 (|x-2|+|x-3|+|x-4|)dx$

2. $\int_{-1}^{2} |x^3 - x| \, dx$

3. $\int_{-\pi/2}^{\pi/2} [\sin|x| - \cos|x|] \, dx$

Answers

Definite Integral as a limit of sum.

LEVEL I

1. 6 2. 8

LEVEL II

1. 6 2. 12

LEVEL III

1. $\frac{35}{6}$ 2. $\frac{14}{3}$

Definite Integrals based upo types of Indefinite Integrals.

LEVEL I

1. $\frac{1}{5}\log 6+\frac{3}{\sqrt{5}}\tan^{-1}\sqrt{5}$

2. $\frac{64}{231}$ 3. $\log\frac{3}{2}-9\log\frac{5}{4}$

LEVEL II

1. $5-10\log\left(\frac{15}{8}\right)+\frac{25}{2}\log\left(\frac{6}{5}\right)$

2. $\frac{e^2}{4}\left(e^2-2\right)$

LEVEL III

1. $\frac{1}{40}\log 9$ 2. $2\sin^{-1}\frac{\sqrt{3}-1}{2}$

Properties of definite Integral

LEVEL I

1. $\pi/4$ 2. $\pi/4$ 3. 1 4. 0

LEVEL II

1. $\frac{\pi}{2\sqrt{2}}\log\left(\sqrt{2}+1\right)$ 2. $\frac{\pi^2}{4}$ 3. $\pi/12$ 4. 0

LEVEL III

1. $-\pi\log 2$ 2. $\pi/8\log 2$ 3. $\pi\log 2$

LEVEL III (Integration of Modulus Functions)

1. $\frac{19}{2}$ 2. $\frac{11}{4}$ 3. 4

APPLICATION OF INTEGRATION

I. Area under simple curve : LEVEL : I

1. Sketch the region of the Ellipse : $\frac{x^2}{25}+\frac{y^2}{16}=1$ and find the area, using integration.
2. Sketch the region : $4x^2 + 4y^2 = 9$ and find the area, using integration.

II. Area of the region enclosed between **Parabola and Line** : LEVEL : II

3. Find the area bounded by the curves $x^2 = 4y$ and the straight line $x = 4y - 2$.

III. Area of the region enclosed between **Ellipse and Line** : Level : II.

4. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the line $\frac{x}{3}+\frac{y}{2}=1$.

IV. Area of the region enclosed between **Circle and Line.** Level : II

5. Find the area of the region $\{(x, y) : x^2 + y^2 \le 4, x + y \ge 2\}$ using integration.
6. Find the area of the region in the first quadrant enclosed by x-axis, the line $x = \sqrt{3}\,y$ and the circle $x^2 + y^2 = 4$.

V. Area of the region enclosed between **Circle and Parabola**. Level : III

7. Using the method of integration, find the area of the region bounded by the parabola $y^2 = 4x$ and the circle. $4x^2 + 4y^2 = 9$.

VI. Area of the region enclosed **between two Circles.** Level : III

8. Find the area of the region enclosed between the two circles $x^2 + y^2 = 1$ and $(x - 1)^2 + y^2 = 1$.

VII. Area of the region enclosed **between two parabolas.** Level : II

9. Find the area of the region included between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, where $a > 0$.

VIII. Area of the triangle when **vertices are given.** Level : III

10. Using the method of integration find the area of the triangle ABC, coordinates of whose vertices. are A (–1, 0), B(1, 3) and C(3, 2).

IX. Area of the triangle when **sides are given,** Level : III

11. Compute the area bounded by the lines $x + 2y = 2$; $y - x = 1$ and $2x + y = 7$.

X. Miscellaneous Question : Level : III

12. *Sketch the graph of* $y = |x + 3|$ *and evaluate the area under the curve* $y = |x + 3|$ above x-axis. and between $x = -6$ to $x = 0$.

XI. 13. Prove that the curves $y^2 = 4x$ and $x^2 = 4y$ divides the area of square bounded by $x = 0$, $x = 4$, and $y = 0$ into three equal parts.

ANSWER : APPLICATION OF INTEGRATIONI

ASSIGNMENTS :

1. 20π sq units,   2. $9/4\pi$ sq units,   3. $9/8$ sq. units.,

4. $3\pi/2 - 3$ sq. units   5. $\pi - 2$ sq units,   6. $\frac{\pi}{3}$ sq. units,

7. $\frac{1}{3\sqrt{2}} + \frac{9\pi}{8} - \frac{9}{8}\sin^{-1}\frac{1}{3}$ sq. units,

8. $2\pi/3 - \frac{\sqrt{3}}{2}$ sq units

9. $16a^2/3$ sq units,   10. 4 sq. units,   11. 6 sq units.

12. 9 sq. units

13. Area of each region = $16/3$ sq. unit

Differential Equation

Order and degree of a differential equation

Level-I

1. Write the order and degree of the following diff. eqn.

$$\left(\frac{d^2y}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y = 0$$

Level-II

2. Write the sum of the order and degtee of the following diff. eqn.

$$\frac{d}{dx}\left\{\left(\frac{dy}{dx}\right)^3\right\} = 0$$

Solutions of a differential equation

Level-I

1. Show that $y = e^{-x} + ax + b$ is a solution of the diff. eqn. $e^x \frac{d^2y}{dx^2} = 1$

Formation of differential equation

Level-II

1. Form the diff. eqn. by eleminating a and b from the eqn.: $y = e^x (a \cos x + b \sin x)$

2. Obtain the differential equation representing the family of parabolas having vertex at the origin and axis along the positive direction of x-axis.

Level-III

3. Find the diff. eqn. of the family of cicles: $(x - a)^2 + (y - b)^2 = r^2$, where r is a fixed constant.

4. Find the diff. eqn. of family of all circles in the second quadrant touching the axes.

Solution of differential equation by the method of variable separable:

Level-I

1, Solve : $\frac{dy}{dx} = 1 + x + y + xy$

2. Solve : $\frac{dy}{dx} = e^{-y} \cos x$, given that $y(0) = 0$

3. Solve : $(1 + x^2)\frac{dy}{dx} - x = \tan^{-1}x$

Homogeneous differential equation of first order and first degree:

Level-II

1. Solve : $(x^2 + xy)dy = (x^2 + y^2)dx$

Level-III

2. Show that the given diff. eqn. is homogeneous and hence solve :

(i) $(x - y)\dfrac{dy}{dx} = x + 2y$ (ii) $ydx + x\log\left(\dfrac{y}{x}\right)dy - 2xdy = 0$

(iii) $xdy - ydx = \sqrt{x^2 + y^2}\,dx$ CBSE 2011

(iv) $x^2ydx - (x^3 + y^3)dy = 0$ (v) $y + x\dfrac{dy}{dx} = x - y\dfrac{dy}{dx}$ AISSCE 2016

(vi) $2ye^{\frac{x}{y}}dx\left[y - 2xe^{\frac{x}{y}}\right] = 0$ (vii) $x^2\dfrac{dy}{dx} = y(x + y)$

Linear differential equation of first order:

Level-I

1. Find the integrating factor of the diff. eqn.: $x\dfrac{dy}{dx} - y = 2x^2$

Level-II

2. Solve the following diff. eqn.:

(i) $\dfrac{dy}{dx} + 2\,y\tan x = \sin x$ (ii) $(1 + x)\dfrac{dy}{dx} - y = e^{3x}(x + 1)^2$

(iii) $x\dfrac{dy}{dx} + y = x\log x$

Level-III

3. Solve the following diff. eqn.:

(i) $\dfrac{dy}{dx} = \cos(x + y)$ (ii) $ye^y dx = (y^3 + 2xe^y)dy$

(iii) $\dfrac{dy}{dx} + \dfrac{4x}{x^2 + 1}y = -\dfrac{1}{(x^2 + 1)^3}$ (iv) $(x + 2y^2)\dfrac{dy}{dx} = y$, *given*, $y(2) = 1$

(v) $xdy + (y + 2x^2)dx = 0$

(vi) $xdy + (y - x^3)dx = 0$

ANSWERS

Order and Degree of differential equation

Level I, Q. 1 Order is 2, Degree is 2

Level II, Q. 2 $\therefore$ order is 2, degree 1, sum is 3

Solutions of a differential equation

Level I, Q. 1 $y = e^{-x} + ax + b$ is a solution of $e^{x}\frac{d^2y}{dx^2} = 1$

Formation of differential equation:

Leve II. Q. 1 $\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 2y$

Q. 2 $\frac{dy}{dx} = \frac{y}{2x}$

Level III, Q. 3 $\left[1+\left(y'\right)^2\right]^3 = r^2\left(y''\right)^2$

Q. 4 $(x^2 + y^2)(1 - y)^2 + 2(x + yy')(1 - y')(y - x) + (x + yy')^2 = 0$

Variable Separable:

Level I, Q. 1 $\log(1 + y) = x + \frac{x^2}{2} + C$

Q. 2 $e^y = \sin x + 1$

Q. 3 $y = \frac{1}{2}\log\left(1 + x^2\right) + \frac{1}{2}\left(\tan^{-1} x\right)^2 + C$

Homogeneous Differential Equation:

Level III, Q. 2(v) $C = \sqrt{x^2 - 2xy - y^2}$

Q. 2(vi) $\log x = -2e^{\frac{x}{y}} - \log\frac{x}{y} + K$

Q. 2(vii) $x^2\frac{dy}{dx} = y(x + y)$

Linear Differential Equation

Level I, Q. 1 $\frac{y}{x} = 2x + k$

Level II, Q. 2(i) $y\sec^2 x = \sec x + K$

Q. 2(ii) $\frac{y}{1+x} = \frac{(1+x)e^{3x}}{3} - \frac{e^{3x}}{9} + k$

Q. 2(iii) $xy = \frac{1}{2}x^2\log x - \frac{1}{4}x^2 + K$

Level Ill, Q. 3(i) $-\cot(x + y) + \operatorname{cosec}(x + y) = x + k$

Q. 3(ii) $x = -\frac{y^2}{e^y} + Ky^2$

Q. 3 (iii) $y(x^2 + 1)^2 = -\tan^{-1} x + K$

Q. 3(iv) $x = 2y^2 + ky$

Q. 3(v) $xy + \frac{2}{3}x^3 = K$

Q. 3(v) $xy = \frac{1}{4}x^4 + K$

VECTOR ALGEBRA

Vectors & Scalars, Magnitude of vector, Unit vector, Direction cosines

Level-I

1. Find the unit vector in the direction of the vector $2\hat{i} - 3\hat{j} + 6\hat{k}$.
2. Write the vector in the direction of the vector $\hat{i} - 2\hat{j} - 2\hat{k}$ that has magnitude 5 units
3. Find the unit vector in the direction of the sum of vectors $\hat{i} - \hat{j} + \hat{k}$, $2\hat{i} + \hat{j} - 3\hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$
4. Find a vector in the opposite direction of the vector $\vec{a} = 3\hat{i} - 2\hat{j} - 9\hat{k}$ that has magnitude 5
5. If l, m, n are the dc's of a vector, write a unit vector parallel to it

Level-II

6. Find a vector of magnitude 5 perpendicular to the vectors $\hat{i} + \hat{j} - \hat{k}$ and $\hat{i} - \hat{j} - \hat{k}$
7. Find the unit vector perpendicular to vectors $\hat{i} - \hat{j} + \hat{k}$ and $2\hat{i} + \hat{j} + \hat{k}$
8. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.
9. If $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$, and $\vec{c} = 2\hat{i} + 3\hat{k}$, find a vector of magnitude 6 units in the direction of $2\vec{a} - \vec{b} + \vec{c}$
10. Find the dc's of the line parallel to the vector $\hat{i} - 2\hat{j} + 2\hat{k}$

Level-III

11. If a line makes angles α, β and γ with co–ordinate axes x, y and z resp. then prove $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$
12. Find the value of p if the vector $3\hat{i} + 2\hat{j} + 9\hat{k}$ is parallel to vector $\hat{i} + p\hat{j} + 3\hat{k}$
13. Find a vector of magnitude 6 parallel to a line whose dc's are $\frac{1}{3}, \frac{2}{3}, \frac{-2}{3}$

Position Vectors , Collinear vectors

Level-I

14. Find the position vector of the mid point of line segment joining points $5\hat{i} - \hat{j} + 4\hat{k}$ and $3\hat{i} + 3\hat{j} + 2\hat{k}$
15. In a triangle sides AB and AC are represented by vectors $2\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 3\hat{j} + 5\hat{k}$, find the vector represented by BC
16. Show that points (1,0), (6,0) and (0,0) are collinear using vectors

Level-II

17. Find the position vector of a point which divides the join of points with position vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ internally in the ratio 2 : 1

18. Find the position vector of a point R which divides the join of points P & Q with position vectors $\vec{a} - 2\vec{b}$ and $2\vec{a} + \vec{b}$ resp. externally in the ratio 1: 2. Also show that P is the mid point of line segment RQ.

Level-III

19. Using vectors show that points (3,2,1),(5,5,2) and (–1,–4,–1) are collinear.

Dot Product of two vectors

Level-I

20. Find $\vec{a}\,.\,\vec{b}$ if $\vec{a} = 3\hat{i} - \hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + 3\hat{k}$

21. If $|\vec{a}| = \sqrt{3}, |\vec{b}| = 2$ and $\vec{a}\,.\,\vec{b} = \sqrt{6}$, find angle between $\vec{a}$ & $\vec{b}$

Level-II

22. The dot product of a vector with vectors i–3j, i–2j and i+j+4k are resp. 0,5 and 8, find the vector.

23. If $\vec{a}$ and $\vec{b}$ are the vectors such that $|\vec{a}\,.\,\vec{b}| = |\vec{a} \times \vec{b}|$, find the angle between $\vec{a}$ & $\vec{b}$

24. If $\vec{a} = 2\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j}$ such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, find the value of λ.

Level-III

25. If $\vec{a}$ and $\vec{b}$ are two unit vectors inclined at an angle θ then prove that $\sin\frac{\theta}{2} = \frac{1}{2}|\vec{a} - \vec{b}|$

26. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ then find the angle between $\vec{a}$ and $\vec{b}$

27. Find the value of p if vectors $3\hat{i} - 2\hat{j} + 3\hat{k}$ and $p\hat{i} - 4\hat{j} + 8\hat{k}$ are orthogonal.

28. Find $|\vec{x}|$ if for unit vector $\vec{a}$, $(\vec{x} - \vec{a}).(\vec{x} + \vec{a}) = 15$

29. If $\vec{a} = 5\hat{i} - \hat{j} + 7\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \mu\hat{k}$, find μ such that $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular to each other

30. Show that the vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ form the sides of a right triangle.

31. If $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$, find a vector $\vec{d}$ which is perpendicular to $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 18$

32. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular vectors, prove that $\vec{a} + \vec{b} + \vec{c}$ is equaly inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$

Projection of Vector

Level-I

33. Find the projection of vector $\vec{a}$ on $\vec{b}$ if $\vec{a}.\vec{b} = 8$ and $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

34. Write the projection of vector $\hat{i} - \hat{j}$ on $\hat{i} + \hat{j}$

35. Find the projection of vector $\hat{i} + 3\hat{j} - 7\hat{k}$ on $7\hat{i} - \hat{j} + 8\hat{k}$

Level-II

36. Three vertices of a triangle are (0,-1,-2), (3,1,4) and (5,7,1), Using vector show that it is a right angle triangle. Also find the other two angles.

37. Show that the angle between two diagonals of a cube is $\cos^{-1}\frac{1}{3}$

38. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non—zero and non–coplanar vectors then prove that $\vec{a} - 2\vec{b} + 3\vec{c}$, $-3\vec{b} + 5\vec{c}$ and $2\vec{a} + 3\vec{b} - 4\vec{c}$ are co-planer.

39. If a unit vector $\vec{a}$ makes angle $\frac{\pi}{4}$ with $\hat{i}$, $\frac{\pi}{3}$ with $\hat{j}$ and an acute angle θ with $\hat{k}$, then find the component vector of $\vec{a}$ and angle θ.

40. $\vec{a} = 3\hat{i} - \hat{j}$ and $\vec{b} = 2\hat{i} + \hat{j} - 3\hat{k}$ then express vector $\vec{b} = \vec{b}_1 + \vec{b}_2$ where $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$

41. Show that the points A, B and C with position vectors $3\hat{i} - 4\hat{j} - 4\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} - 3\hat{j} - 5\hat{k}$ resp. form the vertices of a right triangle.

42. If $\vec{a}$ & $\vec{b}$ are unit vectors inclined at an angle θ then prove that

(i) $\sin\frac{\theta}{2} = \frac{1}{2}\left|\vec{a} - \vec{b}\right|$ (ii) $\tan\frac{\theta}{2} = \frac{\left|\vec{a} - \vec{b}\right|}{\left|\vec{a} + \vec{b}\right|}$

Cross Product of two vectors

Level-I

43. If $|\vec{a}| = 3$, $|\vec{b}| = 5$ and $\vec{a}.\vec{b} = 9$, find $\vec{a} \times \vec{b}$

44. Find $\vec{a} \times \vec{b}$. if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} + 2\hat{k}$

45. Find p if $(2\hat{i} + 6\hat{j} + 27\hat{k}) \times (\hat{i} + 3\hat{j} + p\hat{k}) = 0$

Level-II

46. Show that $(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b}) = 2(\vec{a} \times \vec{b})$

47. Find the angle between two vectors $\vec{a}$ & $\vec{b}$ if $|\vec{a}| = 3$, $|\vec{b}| = 4$ and $\vec{a} \times \vec{b} = 6$

48. Vectors $\vec{a}$ & $\vec{b}$ are such that $|\vec{a}| = \sqrt{3}, |\vec{b}| = \frac{2}{3}$ and $(\vec{a} \times \vec{b})$ is a unit vector, find the angle between $\vec{a}$ & $\vec{b}$.

49. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a}.\vec{b} = \vec{a}.\vec{c} = 0$ and angle between vectors $\vec{b}$ & $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a} = \mp 2(\vec{a} \times \vec{b})$.

Level-III

50. Find the value of the following $\hat{i}.(\hat{j} \times \hat{k}) + \hat{i}.(\hat{j} \times \hat{k}) + \hat{k}.(\hat{i} \times \hat{j})$

51. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and b = $\hat{j} - \hat{k}$, find $\vec{a}$ vector $\vec{c}$ such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a}.\vec{c} = 3$

52. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$, show that $\vec{a} - \vec{d}$ is parallel to $\vec{b} - \vec{c}$, where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$

53. Express vector = $2\hat{i} - \hat{j} + 3\hat{k}$ as the sum of two vectors with one parallel and other perpendicular to $2\hat{i} + 4\hat{j} - 2\hat{k}$.

Area of triangle and Prallelogram

Level-I

54. Find the area of the parallelogram whose adjacent sides are vectors $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$.

55. Find the area of triangle whose vertices are (1,1,1), (1,2,3) and (2,3,1) using vectors

Level-II

56. Find the area of parallelogram whose diagonals are the vectors $3\hat{i} + \hat{j} - 2\hat{k}$ and $\hat{i} - 3\hat{j} + 4\hat{k}$.

57. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are position vectors of vertices of triangle ABC then show that the area of the triangle is $= \frac{1}{2}\left|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}\right|$

Scalar triple product

Level-I

58. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = -\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + \hat{j} + 2\hat{k}$ then find $\vec{a}.\left(\vec{b}\times\vec{c}\right)$

59. Find the volume of parallelopiped whose edges are represented by vectors $\hat{i} + \hat{j} + \hat{k}$, $\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - \hat{k}$

60. Evaluate $\vec{a}.\left(\vec{b}\times\vec{c}\right) + \vec{a}.\left(\vec{a}\times\vec{c}\right) + \vec{a}.\left(\vec{b}\times\vec{a}\right)$

61. Show that $\left[\vec{a}+\vec{b}\;\vec{b}+\vec{c}\;\vec{c}+\vec{a}\right] = 2\left[\vec{a}\;\vec{b}\;\vec{c}\right]$

Level-II

62. Show that the vectors $3\hat{i} - 4\hat{j} + 5\hat{k}$, $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - 3\hat{k}$ are coplanar.

63. Find λ if the vectors $\hat{i} + 3\hat{j} + \hat{k}$, $2\hat{i} - \hat{j} - \hat{k}$ and $\lambda\hat{j} + 3\hat{k}$ are coplanar.

64. If vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar then prove that vectors $\vec{a}+\vec{b}$, $\vec{b}+\vec{c}$, and $\vec{c}+\vec{a}$ are also coplanar.

Level-III

65. Show that the four points $4\hat{i} + 5\hat{j} + \hat{k}$, $-\hat{j} - \hat{k}$, $3\hat{i} + 9\hat{j} + 4\hat{k}$ and $-4\hat{i} + 4\hat{j} + 4\hat{k}$ are coplanar.

66. Find the value of x if the points A(4,1,2), B(5,x,6), C(5,1,-1) and D(7,4,0) are coplanar.

Answer

1.	$2\hat{i} - 3\hat{j} + 6\hat{k}$	2.	$5(\hat{i} - 2\hat{j} - 2\hat{k}) / 3$
3.	$(4\hat{i} + \hat{j} - \hat{k}) / \sqrt{18}$	4.	$-5(3\hat{i} - 2\hat{j} + 9\hat{k}) / \sqrt{94}$
5.	$l\hat{i} + m\hat{j} + n\hat{k}$	6.	$5(\hat{j} - \hat{k}) / \sqrt{2}$
7.	$-2\hat{i} + \hat{j} + 3\hat{k} / \sqrt{14}$	8.	-------
9.	$6(2\hat{i} + 7\hat{j} + 4\hat{k}) / \sqrt{69}$	10.	1/3, –2/3, 2/3
11.	-------	12.	P = 2/3
13.	$2\hat{i} + 4\hat{j} - 4\hat{k}$	14.	$4\hat{i} + \hat{j} + 3\hat{k}$
15.	$-\hat{i} + 4\hat{j} + 3\hat{k}$	16.	---------
17.	$(-\hat{i} + 4\hat{j} + \hat{k}) / 3$	18.	$-5\vec{b}$
19.	----------	20.	9
21.	$\pi/4$	22.	$15\hat{i} + 5\hat{j} - 3\hat{k}$
23.	$\pm \pi/4$	24.	4
25.	-----	26.	$\pi/2$
27.	–32/3	28.	4
29.	$\pm\sqrt{73}$	30.	-------
31.	$64\hat{i} - 2\hat{j} - 28\hat{k}$	32.	-----
33.	8/7	34.	0
35.	$60/\sqrt{114}$	36.	--------
37.	---------		
38.	---------	39.	$i/\sqrt{2}$, $j/2$, $k/2$, $\pi/4$
40.	$(3\hat{i} - \hat{j}) / 2, (\hat{i} + 3\hat{j} - 6\hat{k}) / 2$	41.	---------
42.	-------	43.	12
44.	$28\hat{i} + 19\hat{j} - 23\hat{k}$	45.	27/2

46. ----------
47. $\pm\pi/6$
48. $\pi/3$
49. ---------
50. 2
51. $(5\hat{i} + 2\hat{j} + 2\hat{k}) / 3$
52. ------------
53. $-(2\hat{i} + 4\hat{j} - 2\hat{k}) / 4, 5(\hat{i} + \hat{k}) / 2$
54. $\sqrt{300} = 10\sqrt{3}$
55. $\frac{1}{2}\sqrt{21}$
56. $5\sqrt{3}$
57. ---------
58. -10
59. 4
60. 1
61. --------
62. -----------
63. 7
64. ---------
65. -----------
66. 4

Three Dimensional Geometry

Direction Ratios and Direction Cosines

Level-I

1. Write the direction cosines of the line joining the points (1, 0, 0) and (0, 1, 1).
2. Find the direction cosines of the line passing through the points (–2, 4, –5) and (1, 2, 3).

Level-II

1. Write the direction cosiness of a line parallel to the line $\frac{3-x}{3} = \frac{y+2}{-2} = \frac{z+2}{6}$
2. Write the direction ratios of a line parallel to the line $\frac{5-x}{3} = \frac{y+7}{-2} = \frac{z+2}{6}$
3. If the equation of line AB = $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+6}{3}$. Find the direction cosines.
4. Find the direction cosines of a line, passing through origin and lying in the first octant, making equal angles with the three co-ordinate axes.

Cartesian and vector equation of a line in space and conversion of one into another form.

Level-I

1. Write the vector equation of the line $\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$
2. Write the equation of a line parallel to the line $\frac{x-2}{-3} = \frac{y+3}{2} = \frac{z+5}{6}$ and passing through the point (1,2,3).
3. Express the equation of the line $\vec{r} = (\hat{i} - 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + \hat{j} + 2\hat{k})$ in the Cartesian form.

Co-planer and skew lines

Level-II

1. Find whether the lines $r = (\hat{i} - \hat{j} - \hat{k}) + \lambda(2\hat{i} + \hat{j})$ and $r = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$ intersect or not. If intersecting, find their point of intersection.
2. Show that the four points (0,–1,–1), (4,5,1),(3,9,4) and (–4,4,4) are coplanar.
3. Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect.

Level-III

1. Show that the lines $\frac{x+3}{-3}=\frac{y-1}{1}=\frac{z-5}{5}$ and $\frac{x+1}{-1}=\frac{y-2}{2}=\frac{z-5}{5}$ are coplanar.

2. The point A(4,5,10), B(2,3,4) C(1,2,–1) are three vertices of a parallelogram ABCD. Find the vector equation of the side AB and BC of D.

Shortest distance between two lines :

Level-II

1. Find the shortest distance between the lines l_1 and given by the following :

   (a) $l_1 : \frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-1}{1}$, $l_2 : \frac{x-2}{2}=\frac{y+1}{1}=\frac{z+1}{2}$

   (b) $l_1 : \vec{r}=(\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-3\hat{j}+2\hat{k})$

   $l_2 : \vec{r}=(4+2\mu)\hat{i}+(5+3\mu)\hat{j}+(6+\mu)\hat{k}$

2. Show that the lines $\frac{x-1}{1}=\frac{y-2}{-1}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ intersect.

3. Find the vector equation of the line parallel to the line $\frac{x-1}{5}=\frac{3-y}{2}=\frac{z+1}{4}$ and passing through (3,0,–4). Also, find the distance between the lines.

Angles between two lines

Level-II

1. Find the angle between the lines whose direction ratios are $\langle 1,1,2\rangle$ and $\langle \sqrt{3}-1,-\sqrt{3}-1,4\rangle$

2. Find the value of P, such that the lines $\frac{x}{1}=\frac{y}{3}=\frac{z}{2p}$ and $\frac{x}{-3}=\frac{y}{5}=\frac{z}{2}$ are perpendicular to each other.

3. A line makes angles α, β, γ, δ with the four diagonals of a cube, prove that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}$

4. Find the image ot the point (1,–2, 1) in the line $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z+3}{2}$.

Cartesian and Vector equation of a plane in space and conversion of one into another form :

Level-I

1. Find the equation of a plane passing through the origin and perpendicular to x-axis.

2. Find the equation of plane with intercepts 2, 3, 4 on the x, y, z -axis respectively.

3. Find the direction cosines of the unit vector perpendicular to the plane $\vec{r}\cdot\left(6\hat{i}-3\hat{j}-2\hat{k}\right)+1=0$ passing through the origin.

4. Find the cartesian equation of the plane,
$\vec{r}\cdot\left(\hat{i}+\hat{j}-\hat{k}\right)=2$

Level-II

1. Find the vector and cartesian equations of the plane which passes through the point (5, 2, –4) and perpendicular to the line with direction ratio 2, 3, –1.

2. Find the vector equation of a plane which is at a distance of 7 units from the origin and normal to the vector $3\hat{i}+5\hat{j}-6\hat{k}$.

3. Find the vector and cartesian equations of the planes that passes through the point (1, 0, –2) and the normal to the plane is $\hat{i}+\hat{j}+\hat{k}$.

Angles between (a) Two planes (b) Line and plane

Level-I

1. Find the Angle between the line $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-3}{2}$ and the plane $3x + 4y + z + 5 = 0$.

2. Find the value of λ such that the line $\frac{x-2}{9}=\frac{y-1}{\lambda}=\frac{z+3}{-6}$ is perpendicular to the plane $3x - y - 2z = 7$

3. Find the angles between the planes whose vector equations are
$\vec{r}\cdot\left(2\hat{i}+2\hat{j}-3\hat{k}\right)=5$ and $\vec{r}\cdot\left(3\hat{i}-3\hat{j}+5\hat{k}\right)=3$

4. Find the angle between line $\frac{x+1}{2}=\frac{y}{3}=\frac{z-3}{6}$ and the plane $10x + 2y - 11z = 3$.

Distance of a point from a plane.

Level-I

1. Write the distance of plane $2x - y + 2z + 1 = 0$ from the Origin.

2. Find the distance of a point (2, 5, –3) from the plane $\vec{r}\cdot\left(6\hat{i}-3\hat{j}+2\hat{k}\right)=4$.

3. Find the distance of the plane $2x - y + 2z + 1 = 0$ from Origin

Level-II

1. Find the distance of the point (3, 4, 5) from the plane x + y + z = 2 measured parallel to the line 2x = y = z.

2. Find the distance between the point P (6, 5, 9) and the plane determined by the points A (3, –1, 2), B (5, 2, 4) and C (–1, –1, 6)

3. Find the distance of the point (–1, –5, –10) from the point by intersection of the line $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda\left(3\hat{i} + 4\hat{j} + 2\hat{k}\right)$ and the plane $\vec{r}\cdot\left(\hat{i} - \hat{j} + \hat{k}\right) = 5$.

Level-III

1. Find the co-ordinates of the foot of the perpendicular and the perpendicular distance of the point (1, 3, 4) from the plane 2x – y + z + 3 = 0. Find also the image of the point in the plane.

2. Find the distance of the point P(6, 5, 9) from the plane determined by the points A(3, –1, 2) B(5, 2, 4) and C(–1, –1, 6)

3. Find the equation of the plane containing the lines $\vec{r} = \hat{i} + \hat{j} + \lambda\left(\hat{i} + 2\hat{j} - \hat{k}\right)$ and $\vec{r} = \hat{i} + \hat{j} + \mu\left(-\hat{i} + \hat{j} - 2\hat{k}\right)$. Find the distance of this plane from origin and also from the point (1, 1, 1).

Equation of a plane through the intersection of two planes :

Level-II

1. Find the equation of plane passing through the point (1, 2, 1) and perpendicular to the line joining the point (1, 4, 2) and (2, 3, 5). Also find the perpendicular distance of the plane from the origin.

2. Find the equation of the plane which is perpendicular to the plane 5x + 3y + 6z + 8 = 0 and which contains the line of intersection of the planes x + 2y + 3z – 4 = 0 and 2x + y – z + 5 = 0

3. Find the equation of the plane that contains the point (1, –1, 2) and is perpendicular to each of the planes 2x + 3y – 2z = 5 and x + 2y – 3z = 8

Level-III

1. Find the equation of the plane passing through the point (1, 1, 1) and containing the line $\vec{r} = \left(-3\hat{i} + \hat{j} + 5\hat{k}\right) + \lambda\left(3\hat{i} - \hat{j} - 5\hat{k}\right)$. Also show that the plane contains the line $\vec{r} = \left(-\hat{i} + 2\hat{j} + 5\hat{k}\right) + \lambda\left(\hat{i} - 2\hat{j} - 5\hat{k}\right)$.

2. Find the equation of the plane passing through the point (1, 1, –1) and perpendicular to the planes x + 2y + 3z – 7 = 0 and 2x – 3y + 4z = 0.

Foot of perpendicular and image with respect to line and plane

Level-II

1. Find the foot of the perpendicular from P (1, 2, 3) on the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{2}$.
   Also obtain the equation of the plane containing the line and the point (1, 2, 3).
2. Prove that the image of the point (3, –2, 1) in the plane 3x – y + 4z = 2 lies on the plane x + y + z + 4 = 0.

Level-III

1. The foot of perpendicular from the origin to the plane is (12, –4, 3). Find the equation of the plane.
2. Find the co-ordinate of the foot of the perpendicular and the perpendicular distance of the point P(3, 2, 1) from the planes 2x – y + z + 1 = 0. Find also, the image of the point in the plane.

ANSWERS

Direction Ratios and Direction cosines :

Level-I

1. $\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ 2. $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$

LEVEL – II

1. $\frac{-3}{7}, \frac{-2}{7}, \frac{6}{7}$ 2. (–3, –2, 6)
3. $\frac{2}{\sqrt{14}}, \frac{-1}{\sqrt{14}}, \frac{3}{\sqrt{+14}}$ 4. $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$

Cartesian and vector equation of a line in space and conversion of one into another form :

Level-I

1. $\vec{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \lambda(3\hat{i} + 7\hat{j} - 2\hat{k})$ 2. $\left(\frac{x-1}{-3}, \frac{y-2}{2}, \frac{z-3}{6}\right)$
3. $\frac{x-1}{2} = \frac{y+2}{1} = \frac{z-1}{2} = \lambda$

Co-planar and skew lines :

Level-II

1. Lines are intersecting and point of intersection is (3, 0, –1).

Level-III

2. Equation of AB is $\vec{r} = (4\hat{i} + 5\hat{j} + 10\hat{k}) + \lambda(2\hat{i} + 2\hat{j} + 6\hat{k})$.
   Equation of BC is $\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + \mu(\hat{i} + \hat{j} + 5\hat{k})$.

Shortest distance between two lines :

Level-II

1.(a) $\frac{3\sqrt{2}}{2}$ units (b) $\frac{3}{\sqrt{19}}$ units

4. Vector equation $\vec{r} = \left(3\hat{i} - 4\hat{k}\right) + \lambda\left(5\hat{i} - 2\hat{j} + 4\hat{k}\right)$ and distance $= 7.75$ units.

Angles between two lines :

Level-II

1. 60° 2. P = –3 4. $\left(\frac{39}{7}, \frac{-6}{7}, \frac{-37}{7}\right)$

Cartesian and vector equation of a plane in space and conversion of one into another form.

Level-I

1. $x = 0$ 2. $12x + 4y + 3z = 12$

3. $\frac{-6}{7}, \frac{3}{7}, \frac{2}{7}$ 4. $x + y - z = 2$

1. $2x + 3y - z = 20$ 2. $\vec{r} \cdot \frac{\left(3\hat{i} + 5\hat{j} - 6\hat{k}\right)}{\sqrt{70}} = 7$

3. $\left[\vec{r} - \left(\hat{i} - 2\hat{k}\right)\right] \cdot \left(\hat{i} + \hat{j} - \hat{k}\right) = 0$, $x + y - z = 3$.

Angles between (a) Two planes (b) Line an plane

Level-I

1. $\sin^{-1}\left(\frac{7}{2\sqrt{91}}\right)$ 2. $\lambda = -3$

3. $\cos^{-1}\frac{15}{\sqrt{731}}$ 4. $\sin^{-1}\frac{8}{21}$

Distance of a point from a plane

Level-I

1. $\frac{1}{3}$ 2. $\frac{13}{7}$ 3. $\frac{1}{3}$

Level-II

1. 6 units 2. $\frac{3\sqrt{34}}{17}$ 3. 13

Level-III

1. Foot of perpendicular (–1, 4, 3), Image (–3, 5, 2)

   Distance = $6\sqrt{2}$ units.

2. $3x - 4y + 3z - 19 = 0$

3. $x + y - z - 2 = 0$, $\frac{2}{\sqrt{3}}$ units, $\frac{1}{\sqrt{3}}$ units

Equation of a plane through the intersection of two planes :

Level-II

1. $x - y + 3z - 2 = 0$, $\frac{2\sqrt{11}}{11}$

2. $51x + 15y - 50z + 173 = 0$

3. $5x - 4y - z = 7$.

Level-III

1. $x - 2y + z = 0$
2. $17x + 2y - 7z - 26 = 0$

Foot of perpendicular and image with respect to a line and plane.

Level-II

1. (3, 5, 9)
2. Image of the point = (0, –1, –3)

Level-III

1. $12x - 4y + 3z = 169$

2. (–1, 4, –1)

LINEAR PROGRAMMING

LPP and its Mathematical Formulation

Level-I

1. A dietician wishes to mix two types of foods in such a way that vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food T contains 2 units/kg of vitamin A and 1 unit/kg of vitamin C. Food "II" contains 1 unit/kg of vitamin A and 2 units/kg of vitamin C, It costs Rs 50 per kg to purchase Food T and Rs 70 per kg to purchase Food IF. Formulate this problem as a linear programming problem.

Graphical method of solving LPP (bounded and unbounded solutions)

Level-I

Solve the following Linear Programming Problems graphically :

1. Minimise $Z = -3x + 4y$ subject to $x + 2y \leq 8$, $3x + 2y \leq 12$, $x \geq 0$, $y \geq 0$.
2. Maximise $Z = 5x + 3y$ subject to $3x + By \leq 15$, $5x + 2y \leq 10$, $x \geq 0$, $y \geq 0$.
3. Minimise $Z = 3x + 5y$ such that $x + 3y \geq 3$, $x + y \geq 2$, $x, y \geq 0$.

Diet Problem

Level-II

1. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 calories. Two foods X and Y are available at a cost of Rs. 4 and Rs, 3 per unit respectively. One unit of the food X contains 200 units of vitamins, 1 unit of mineral and 40 calories, whereas one unit of food Y contains 100 units of vitamins, 2 units of minerals and 40 calories. Find what combination of X and Y should be used to have least cost ? Also find the least cost.
2. Every gram of wheat provides 0.1 g of proteins and 0.25 g of carbohydrates. The corresponding values for rice are 0.05 g and 0.5 g respectively. Wheat costs Ha. 10 per kg and rice Rs. 20 per kg. The minimum daily requirements of protein and carbohydrates for an average child are 50 gm and 200 gm respectively, In what quantities, should wheat and rice be mixed in the daily diet to provide the minimum daily requirements of protein and carbohydrates at minimum cost?

Manufacturing Problem

Level-II

1. A company manufactures two articles A and B. There are two departments through which these articles are processed: (i) assembly and (ii) finishing departments. The maximum capacity of the assembly department is 60 hours a week and that of the finishing department is 48 hours a week. The production of each article A requires 4 hours in assembly and 2 hours in finishing and that of each unit of B requires 2 hours in assembly and 4 hours in finishing. If the profit is Rs. 6 for each unit of A and Rs. 8 for each unit of B, find the number of units of A and B to be produced per week in order to have maximum profit.

2. A company sells two different product A and B. The two products are produced in 8 common production process which has a total capacity of 300 man hours, It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The demand in the market shows that the maximum number of units of A that can be sold is 70 and that for B is 125, Profit on each unit of A is Rs. 20 and that on B is Rs. 15, How many units of A and B should be produced to maximize the profit? Solve it graphically

LEVEL-III

1. A manufacturer makes two types of cups. A and B. Three machines are required to manufacture the cups and the time in minutes required by each is given below.

Type of Cup	Machines		
	I	II	III
A	12	18	6
B	6	0	9

Each machine is available for a maximum period of 6 hours per day. If the profit on each cup A is 75 paise. and on B it is 50 paise, show that the 15 cups of type A and 30 cups of type B should be manufactured per day to get the profit.

Allocation Problem

Level-II

1. Ramesh wants to invest at most Rs. 70.000 in Bonds A and B. According to the rules, he has to invest at least Rs. 10.000 in Bond A and at least Rs. 30,000 in Bond B. If the rate of interest on bond A is 8% per annum and the rate of interest on bond B is 10% per annum, how much money should he invests to earn maximum yearly income? Find also his maximum yearly income.

2. An oil company requires 12,000, 20,000 and 16,000 barrels of high grade. medium grade and low grade oil respectively. Refinery A produces 100, 800 and 200 barrels per day of high, medium and low grade oil respectively whereas the Refinery B produces 200. 400 and 100 barrels per day respectively. If A costs' Rs. 400 per day and B costs Rs. 300 per day to operate, how' many days should each be run to minimize the cost of requirement ?

Level-III

1. An aeroplane can carry a maximum of 250 passengers. A profit of Rs 500 is made on each executive class ticket and a profit of Rs 350 is made on each economy class ticket. The airline reserves at least 25 seats for executive class. However, at least 3 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit ?

ANSWERS :-

LPP and its Mathematical Formulation

Level - I

1. $Z = 50x + 70y$. $2x + y \geq 8$, $x + 2y \geq 10$. $x, y \geq 0$

Graphical method of solving LPP (bounded and unbounded solutions)

Level -I

1. Minimum $Z = -12$ at $(4, 0)$
2. Maximum $Z - 235/19$ at $(20/19, 45/19)$
3. Minimum $Z = 7$ at $(3/2, 1/2)$

Diet Problem

Level -II

1. Least cost = Rs. 110 at $x = 5$ and $y = 30$
2. Minimum cost = Rs. 6 at $x = 400$ and $y = 200$.

Manufacturing Problem

Level -II

1. Maximum profit is Rs. 120 when 12 units of A and 6 units of B are produced.
2. For maximum profit, 25 units of product A and 125 units of product B are produced and sold.

Allocation Problem

Level -II

1. Maximum annual income = Rs. 6200 on investment of Rs. 40.000 on Bond A and Rs. 30,000 on Bond B.
2. A should run for 60 days and B for 30 days.

Level -III

1. For maximum profit, 62 executive class tickets and 188 economy class ticket should be sold.

PROBABILITY

Conditional probability

Level-I

1. If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ $P(A \cup B) = \frac{7}{11}$ find $P(A \cap B)$, $P(A/B)$ and $P(B/A)$
2. A fair die is rolled. If E = {1, 3. 5}. F = {2. 3} and G = {2. 3. 4. 5} find
   (a) P (E/F)
   (b) P(G/E|
   (c) F((E$\cup$ F)/G)

Level-II

1. A card is drawn at random from a pack of 52 cards and it is found to be a king card. Find the probability that the drawn card is a black card.
2. Two coins are tossed once. Determine P (A/B). where A = Tail appears on one coin and B = One coin shows head,

Level-III

1. Two corns arc tossed. What is the probability of coming up of two heads, if it is known that at least one head comes tip ?
2. A die is thrown twice and the sum of numbers appearing is observed to be 6. What is the conditional probability that the number 4 has appeared at least once.

Multiplication theorem on probability & Independent events

Level-I

1. If A and B are independent event and $P(A \cap B) = 1/8$, $P(A' \cap B') = 3/8$, find P(A) and P(B),

Level-II

1. A bag contains 5 white. 7 red, and 3 black balls. If three balls are drawn one by one without replacement, find the probability that none is red.
2. The probability of a hitting a target is 3/7 and that of B hitting is 1/3. They both fire at the target. Find the probability that (a) at least one of them will hit the target (b) only one of them will hit the target.

Level-III

1. Two integers are selected from integers 1 to 11. If the sum is even, find the probability that both the numbers are odd.
2. A speaks the truth to 60% cases and B in 70%; what is the probability that they will agree in stating the same fact ?

Bayes' theorem

Level-I

1. Bag A contains 2 white and 8 red balls, and bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bag and it is found to be red. Find the probability that it was drawn from bag B.

2. A card from a pack of 52 cards is lost. From the remaining cards of the pack, two card are dawn and are found to be both spades. Find the probability of the lost card being a spade.

3. A factory has two machines A and B. past records shows that machine a produced 60% of the items of output arid machine B produced 40% of the items. Further, 2% of the items produced by machine A and 1% produced by machine B were defective. All the items are put into stockpile and then one item is chosen at random from this and is found to be defective. What is the probability that it was produced by machine B ?

4. A speak truth in 60% of the cases and B speaks truth in 90% of the cases. In what percentage of the cases they contradict each other in starting the same fact.

Level-II

1. A man is known to speak truth 3 out 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

2. Suppose a girl throws a die. If she gets a 5 or 6, she tosses a coin three times and notes the number of heads. If she gets a 1,2,3. or 4, she tosses a coin once and notes whether a head or tail is obtained. If she obtained exactly one head; what is the probability that she threw a 1, 2, 3, or 4 with the die ?

3. In an examination, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is 1/3 and the probability that he copies the answer is 1/6. The probability that his answer is correct given that he copied it is 1/8. The probability that his answer is correct given that he guessed it is (1/4) . Find the probability that he knew the answer to the question given that he correctly answered it.

Level-III

1. A letter is known to have come either from TATANAGAR or CALCUTTA. On the Envelop just two consecutive letters TA are visible. What is the probability that the letter has come from (1) CALCUTTA (II) TATANAGAR

2. A bag contains 4 balls, two balls are drawn at random and are found to be white. What is the probability that all balls are white?

3. Suppose that the reliability of a HIV test is specified as follows:

   Of people having HIV, 90% of the test detected the diseases but 10% got undetected. Of people free of HIV, 99% of the test are judged HIV negative but 1% are diagnosed as showing HIV positive. From a large population of which only 0.1% have HIV. one person is selected at random, given the HIV test and the Pathologist reports him/her as HIV +ve. What is the probability that the person actually has HIV ?

Level-III

4. Bag I contains 3 red and 4 black balls and another bag II contains 5 red and 6 black balls. One ball is drawn at random from bag I and without noticing its colour, it Is put into the bag II. Then a ball is drawn from bag II find the Probability of getting a red ball.

Radom variables & probability distribution mean, variance and standard deviation of random variables

Level-I

1. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability of the number of aces

2. A die is tossed once. If the random variable X is defined as

$$X = \begin{cases} 1. \text{ if the die results in an even no.} \\ 0, \text{ if the die results in an odd no.} \end{cases}$$

Then find the mean and variance of X,

3. 3 defective bulbs are mixed with 7 good ones. Let X be the no, of defective bulbs when 3 bulbs are drawn at random. Find the mean and variance of X.

Level-II

1. Two cards are drawn simultaneously for successively without replacement). For a well shuffled pack of 52 cards. Find the mean and variance of the no. of aces.

2. A random variable X has the following probability distribution

X =	0	1	2	3	4	5	6	7
P(X) =	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find each of the following :

(i) K (ii) $P(x \geq 6)$ (iii) $P(0 < x < 5)$

Level-III

1. A coin is biased so that the head is 3 times as likely to occur as a tail. If the coin is tossed twice. Find the probability distribution of the no. of tails.

2. Two numbers are selected at random (without replacement] from the first six positive-integers. Let X denotes the larger of the Two numbers obtained. Find the probability distribution of the random variable X. and hence find the mean of the distribution.

BERNOULLI's TRIALS AND BINOMIAL DISTRIBUTION :

Level-II

1. An unbiased coin is tossed 6 times. Find, using binomial distribution, the parabability of getting at least 5 heads.
2. A die is rolled 20 times. Getting a number greater than 4 is a success. Find a mean and variance of the number of success.

ANSWERS

Conditional probability

Level-I

1.. $\frac{4}{11}$, $\frac{4}{5}$ and $\frac{2}{3}$

2. (a) P(E/F) = 1/2

   (b) P(G/E) = 2/3

   (c) $P(E\cup F)/G)$ =3/4

Level-II

1. 1/2
2. 1.

Level-III

1. $\frac{1}{3}$
2. 2/5

Multiplication theorem on probability and independent events

Level-I

1. 1/2 and 1/4

Level-II

1. S/65
2. (a) 13/21 (b) 10/21

Level-III

1. 3/5
2. 27/50

Bayes' theorem

Level-I

1. $\frac{(25)}{(52)}$
2. (0.22)
3. (1/4)
4. (42%)

Level-II

1. (3/8
2. (8/11)
3. 24/29

Level-III

1. 1. (4/11), (7/11)
2. (3/5)
3. 99/1080
4. 19/42

Random Variables & Probability distribution, means,variance and standard deviation.

Level-I

1. P(X = 0) = 144/169. P(X = 1) = 24/169, P(X = 2) = 1/169
2. mean = 1/2 Variance = 1/4
3. mean = 9/10 Variance = 49/100

Level-II

1. mean = 2/30 Variance : 400/2873
2. (i) K = 1/10 (ii) 19/100 (iii) 4/5

Level-III

1.

x	0	1	2		
P(x)	9/16	3/8	1/16		

2.

x	2	3	4	5	6
P(x)	2/30	4/30	6/30	8/30	10/30

Required mean = 4

Bernoulli's Trials and Binomial Distribution :

Level-II

1. 7/64
2. mean : 6.67 variance : 4.44

Question Paper-I

CBSE Previous Years

Mathematics

Time Allowed : 3 hrs. **M.M.: 100 Marks**

General Instruction :

(i) All questions are compulsory.

(ii) Please check that this Question Paper contains 26 Questions.

(iii) Marks for each question are indicated against it.

(iv) Question 1 to 6 in Sector-A are Very Short Answer Type Questions carrying one mark each.

(v) Questions 7 to 19 in Sector-B are Long Answer I Type Questions carrying 4 marks each.

(vi) Question 20 to 26 in Section-C are Long Answer II Type Questions carrying 6 marks each.

(viii) Please write down the serial number of the Question before attempting it.

SECTION-A

Q.1 Find the maximum value of

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin\theta & 1 \\ 1 & 1 & 1+\cos\theta \end{vmatrix}$$

Q.2 If A is a square matrix such that $A^2 = I$, then find the simplified value of

$(A - I)^3 + (A + 1)^3 - 7A$

Q.3 Matrix $A = \begin{vmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & 1 \end{vmatrix}$ is given to be symmetric, find value of a and b.

Q.4 Find the position vector of a point which divides the join of points with position vectors $\vec{a} - 2\vec{b}$ *and* $2\vec{a} + \vec{b}$ externally in the ratio 2:1.

Q.5 The two vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ represent the two sides AB and AC, respectively of a. Find the length of the median through A.

Q.6 Find the vector equation of a plane which is at a distance of 5 units from the origin and its normal vector is $2\hat{i} - 3\hat{j} + 6\hat{k}$

SECTION-B

Q.7 Prove that : $tan^{-1}\frac{1}{5} + tan^{-7}\frac{1}{7} + tan^{-3}\frac{1}{3} + tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

OR

Solve for x:

$2tan^{-1}(\cos x) = tan^{-1}(2\,\text{cosec}\, x)$

Q.8 The monthly incomes of Aryan and Babban are in the ratio 3:4 and their monthly expenditures are in the ratio 5:7. If each saves Rs. 15,000 per month, find their monthly incomes using matrix method. This problem reflects which value ?

Q.9 If $x = a \sin 2t\,(1 + \cos 2t)$ and $y = b\,2t\,(1 - \cos 2t)$, find the values of $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$ and $t = \frac{\pi}{3}$

OR

If $y = x^{x}$, prove that $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}^2\right) - \frac{y}{x} = 0.$

Q. 10 Find the value of p and q for which

$$f(x) = \begin{cases} \frac{1 - \sin^3 x}{3\cos^2 x}, if\ x < \frac{x}{2} \\ p, if\ x = \frac{\pi}{2} \\ \frac{q(1 - \sin x)}{(\pi - 2x)^2}, if\ x > \frac{\pi}{2} \end{cases}$$

is continuous at $x = \frac{\pi}{2}$

Q.11 Show that the equation of normal at any point t on the curve x = 3 cos t – cost and

$y = 3\sin t - \sin^3 t$ is

$4\,(y\cos^3 t - \sin^3 t) = 3\sin 4t.$

Q.12 Find $\int \frac{(3\sin\theta - 2)\cos\theta}{5 - \cos^2\theta - 4\sin\theta}\theta.$

OR

Evaluate $\int_0^x e^{2x}.\sin\left(\frac{\pi}{4} + x\right)dx$

Q.13 Find $\int \frac{\sqrt{x}}{\sqrt{x^3 - a^3}}\,dx$

Q.14 Evaluate $^2\int_{-1} 1x^2 - x\,1dx.$

Q.15 Find the particular solution of the differential equation

$(1 - y^2)(1 + \log x)\,dx + 2xydy = 0$

given that y = 0 when $x = 1$.

Q. 16 Find the general solution of the following differential equation :

$$(1-y^2)(x-e^{tan-1y})\frac{dy}{dx}=0$$

Q.17 Show that the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar if $\vec{a}, \vec{b}, \vec{b}+\vec{c}$ and $\vec{c}+\vec{a}$ are coplanar.

Q.18 Find the vector and Cartesian equations of the line through the point (1, 2, –4) and perpendicular to the two lines.

$\vec{r}=(8\hat{i}-19\hat{j}+10\hat{k})+\lambda(3\hat{i}-16\hat{j}+7\hat{k})$ and

$\vec{r}=(15\hat{i}+29\hat{j}+5\hat{k})+\mu(3\hat{i}+18\hat{j}-5\hat{k})$.

Q.19 Three persons A, B and C apply for a job of Manager in a Private Company. Chances of their selection (A, B and C) are in the ratio 1:2:4. The probabilities that A, B and C can introduce changes to improve profits of the company are 0.8, 0.5 and 0.3, respectively. If the change does not take place, find the probability that it is due to the appointment of C.

OR

A and B throw a pair of dice alternately. A wins the game if he gets a total of 7 and B wins the game if he gets a total of 10. If A starts the game, then find the probability that B wins.

SECTION-C

Q.20 Let $f: N \to N$ be a function defined as $f(x) = 9x^2 + 6x - 5$. Show that $f: N \to S$, where S is the range of f, is invertible. Find the inverse of f and hence find $f^{-1}(43)$ and f^{-1} (163.

Q. 21 Prove that $\begin{vmatrix} yz-x^2 & zx-y^2 & xy-z^2 \\ xz-y^2 & xy-z^2 & yz-x^2 \\ xy-z^2 & yz-x^2 & zx-y^2 \end{vmatrix}$ is divisible by $(x+y+z)$ and hence find the quotient.

OR

Using elementary transformations, find the inverse of the matrix $A=\begin{pmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$ and use it to solve the following system of linear equations :

$8x+4y+3z=19$

$2x+y+z=5$

$x+2y+2z=7$

Q.22 Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4\pi}{3}$. Also find maximum volume in terms of volume of the sphere.

OR

Find the intervals in which f(x) = sin 3x – cos 3x, 0 < x (π, is strictly increasing or strictly decreasing.

Q.23 Using integration find the area of the region $\{(x,y) : x^2 + y^2 \leq 2ax, y^2 \geq ax, x, y \geq 0\}$.

Q.24 Find the coordinate of the point P where the line through A (3, –4, –5) and B (2, –3, 1) crosses the plane passing through three points L (2, 2, 1), M (3, 0, 1) and N (4, –1, 0). Also, find the ratio in which P divides the line segment AB.

Q.25 An run contains 3 white and 6 red balls. Four balls are drawn one by one with replacement from the urn. Find the probability distribution of the number of red balls drawn. Also find mean and variance of the distribution.

Q.26 A manufacturer produces two products A and B. Both the products are processed on two different machines. The available capacity of first machine is 12 hours and that of second machine is 9 hours per day. Each unit of product A requires 3 hours on both machines and each unit of product B requires 2 hours on first machine and 1 hour on second machine. Each unit of product A is sold at Rs. 7 profit and B at a profit of Rs. 4. Find the production level per day for maximum profit graphically.

Question Paper-II

CBSE Previous Years

Mathematics

Time Allowed : 3 hrs. **M.M.: 100 Marks**

SECTION-A

Question numbers 1 to 6 carry 1 mark each.

Q.1 If $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$, find a satisfying $0 < \alpha \frac{\pi}{2}$ when $A + A^T = \sqrt{2}\, I_2$;

where A^T is transpose of A.

Q.2 For what values of k, the system of linear equations
$x + y + z = 2$
$2x + y - z = 3$
$3x + 2y + kz = 4$
has a unique solution?

Q.3 Write the sum of intercepts cut off by the plane $\vec{r}.(2\hat{i} + \hat{j} - \hat{k}) = 5 = 0$ on the three axes.

Q.4 Find λ and μ if

$(\hat{i} + 3\hat{j} + 9\hat{k}) \times (3\hat{i} - \lambda\hat{j} + \mu\hat{k}) = \vec{0}$.

Q.5 If $\vec{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector parallel to the vector $\vec{a} + \vec{b}$.

SECTION-B

Question numbers 7 to 19 carry 4 marks each

Q.6 Solve for x : $\tan^{-1}(x - 1) + \tan^{-1} x + \tan^{-1}(x + 1) = \tan^{-1} 3x$.

OR

Prove that $\tan\left(\frac{6x - 8x^3}{1 - 12x^2}\right) - \tan^{-1}\left(\frac{4x}{1 - 4x^2}\right) = \tan^{-1} 2x;\ |2x| < \frac{1}{\sqrt{3}}$

Q.7 A typist charges Rs. 145 for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are Rs. 180. Using matrices, find the charges of typing one English and one Hindi page separately. However typist charged only Rs. 2 per page from a poor student Shyam for 5 Hindi pages. How much less was charged from this poor boy? Which values are reflected in this problem?

Q.8

$$If\quad f(x) = \begin{cases} \dfrac{\sin(a+1)x + 2\sin x}{x}, & x < 0 \\ 2 & , x = 0 \\ \dfrac{\sqrt{1 + bx} - 1}{x}, & , x > 0 \end{cases}$$

is continuous at $x = 0$, then find the values of a and b.

Q.9 Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left[\frac{6x - 4\sqrt{1-4x^2}}{5}\right]$

Q.10 Find the equation of tangents to the curve $y = x^3 + 2x - 4$, which are perpendicular to line $x + 14y + 3 = 0$.

Q.11 Find : $\int \frac{(2x-5)e^{2x}}{(2x-3)^3}\,dx$

OR

Find : $\int \frac{x^2+x+1}{(x^2+1)(x+2)}\,dx$

Q.12 Find the particular solution of differential equation : $\frac{dy}{dx} = -\frac{x+y\cos x}{1+\sin x}$ given that y = 1 when $x = 0$.

Q.13 Find the particular solution of the differential equation

$2y\, e^{x/y}\, dx + y - 2x\, e^{x/y}\, dy = 0$

given that $x = 0$ when $y = 1$.

Q.14 Show that the four points A (4, 5, 1), B (0, – 1, – 1), C (3, 9, 4) and D (–4, 4, 4) are coplanar.

Q.15 Find the coordinates of the foot of perpendicular drawn from the point A(–1, 8, 4) to the line joining the points B(0, – 1, 3) and C(2, –3, –1). Hence find the image of the point A in the line BC.

Q.16 A bag X contains 4 white balls and 2 black balls, while another bag Y contains 3 white balls and 3 black bass. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag Y.

OR

A and B throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if A starts first.

SECTION-C

Question numbers 20 to 26 carry 6 marks each

Q.17 Three numbers are selected at random (without replacement) from first six positive integers. Let X denote the largest of the three numbers obtained. Find the probability distribution of X. Also, find the mean and variance of the distribution.

Q.18 Let A = R x R and * be a binary operation on A defined by

(a, b) * (c, d) = (a + c, b + d)

Show that * is commutative and associative. Find the identity element for * on A. Also find the inverse of every element (a, b) $\in$ A.

Q.19 Prove that $y = \frac{4\sin\theta}{2+\cos\theta} - \theta$ is an increasing function of θ on $\left[0, \frac{\pi}{2}\right]$.

OR

Show that semi-vertical angle of a cone of maximum volume and given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

Q.20 Using the method of integration, find the area of the triangular region whose vertices are (2, –2), (4, 3) and (1, 2).

Q.21 Find the equation of the plane which contains the line of intersection of the planes

$\vec{r}.(\hat{i} - 2\hat{j} + 3\hat{k}) - 4 = 0$ and

$\vec{r}.(-2\hat{i} + \hat{j} + \hat{k}) + 5 = 0$

and whose intercept on x-axis is equal to that of on y-axis.

Q.22 A retired person wants to invest an amount of Rs. 50,000. His broker recommends investing in two type of bonds 'A' and 'B' yielding 10% and 9% return respectively on the invested amount. He decides to invest at least Rs. 20,000 in bond 'A' and at least Rs. 10,000 in bond 'B'. He also wants to invest at least as much in bond 'A' as in bond 'B'. Solve this linear programming problem graphically to maximise his returns.

Q.23 Using properties of determinants, prove that

$$\begin{vmatrix} (x+y)^2 & zx & zy \\ xz & (z+y)^2 & xy \\ zy & xy & (z+x)^2 \end{vmatrix} = 2xyz\,(x+y+z)^2$$

OR

If $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$ and $A - 6A^2 + 7A^2 + kI_3 = O$ find k.

Question Paper-III

CBSE Previous Years

Mathematics

Time Allowed : 3 hrs. **M.M.: 100 Marks**

General Instruction :

(i) All questions are compulsory.
(ii) Please check that this Question Paper contains 26 Questions.
(iii) Marks for each question are indicated against it.
(iv) Question 1 to 6 in Sector-A are Very Short Answer Type Questions carrying one mark each.
(v) Questions 7 to 19 in Sector-B are Long Answer I Type Questions carrying 4 marks each.
(vi) Question 20 to 26 in Section-C are Long Answer II Type Questions carrying 6 marks each.
(viii) Please write down the serial number of the Question before attempting it.

SECTION-A

Question numbers 1 to 6 carry 1 mark each.

1. Write the direction ratio's of the vector $3\vec{a} + 2\vec{b}$ where $\vec{a} = \hat{i} + j - 2\hat{k}$ and $\vec{b} = 2\hat{i} - 4\hat{i} + 5\hat{k}$.

2. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{i} + 2\hat{k}$ on the vector $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$.

3. Write the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\vec{r}.(\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$.

4. In the interval $\pi/2 < x < \pi$, find the value of x for which the matrix $\begin{pmatrix} 2\sin x & 3 \\ 1 & 2\sin x \end{pmatrix}$ is singular.

5. Find the solution of the differential equation $\frac{dy}{dx} = x^2 e^{-2y}$.

6. Write the integrating factor of the differential equation

$$\sqrt{x}\,\frac{dy}{dx} + y = e^{-2\sqrt{x}}$$

SECTION-B

Question numbers 7 to 19 carry 4 mark each.

7. A trust fund has Rs. 35,000 is to be invested in two different types of bonds. The first bond pays 8% interest per annum which will be given to orphanage and second bound pays 10% interest per annum which will be given to an N.G.O. (Cancer Aid Society). Using matrix multiplication, determine how to divide Rs. 35,000 among two types of bonds if the trust fund obtains an annual total interest of Rs. 3,200. What are the values reflected in this question?

8. Express the matrix $A = \begin{bmatrix} 2 & 4 & -6 \\ 7 & 3 & 5 \\ 1 & -2 & 4 \end{bmatrix}$ as the sum of a symmetric and skew symmetric matrix.

OR

If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, verify that $(AB)^{-1} = B^{-1} A^{-1}$.

9. Using properties of determinants, solve fox x : $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$

10 *Evaluate* $\int_0^{\pi/4} \log(1+\tan x)dx.$

.

11. *Find* $\int \frac{x}{(x^2+1)(x-1)} dx.$

OR

Find $\int_0^{\frac{1}{\sqrt{2}}} \frac{\sin^{-1}}{(1-x^2)^{3/2}} dx.$

12. Four cards are drawn successively with replacement from a well shuffled deck of 52 cards. What is the probability that.

(i) all the four cards are spades?

(ii) only 2 cards are spades?

OR

A pair of dice is thrown 4 times. If getting a doublet is considered a success, find the probability distribution of number of successes. Hence find the mean of the distribution.

13. Prove that $[\vec{a}, \vec{b} + \vec{c}, \vec{d}] = [\vec{a}, \vec{b}, \vec{d}] + [\vec{a}, \vec{c}, \vec{d}]$

14. Find the shortest distance between the following lines :

$\vec{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and $\vec{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

15. Prove that $2\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \sin^{-1}\left(\frac{31}{25\sqrt{2}}\right)$

OR

Solve for x : $\tan^{-1}\left(\frac{1-x}{1+x}\right) = \left(\frac{1}{2}\right)\tan^{-1} x, x > 0$

16. For what value of λ the function defined by $f(x) = \begin{cases} \lambda(x^2+2), if\ x \le 0 \\ 4x+6,\ if\ x > 0 \end{cases}$ is continuous at x = 0 ? Hence check the differentiability of *f(x)* at *x* = 0.

17. If x = ae^t (sin t + cos t) and y = ae^t (sin t – cos t), find $\frac{d^2y}{dx^2}$

18. If y = $Ae^{mx} + Be^{nr}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

19. Find $\int \frac{x+3}{\sqrt{5-4x-2x^2}}\,dx.$

SECTION - C

Question numbers 20 to 26 carry 6 marks each.

20. Show that the relation R in the set A = {1, 2, 3, 4) given by R = {(a, b) : 1a – b1 is divisible by 2} is an equivalence relation. Write all the equivalence classes of R.

21. Find the area of the region {(x, y) : $y^2 \le 4x$, $4x^2 + 4y^2 \le 9$), using integration.

OR

Using integration, find the area enclosed by the parabola $4y = 3x^2$ and the line 2y = 3x + 12.

22. Solve the differential equation

$\left(x \sin^2\left(\frac{y}{x}\right) - y\right) dx + x\,dy = 0$ given $y = \frac{\pi}{4}$ when $x = 1$.

OR

Solve the differential equation $\frac{dy}{dx} - 3y \cot x = \sin 2x$ given $y = 2$ when $x = \frac{\pi}{2}$

23. Find the vector and cartesian equations of the plane passing through the line of intersection of the planes

$\vec{r}.(2\hat{i} + 2\hat{j} - 3\hat{k}) = 7$, $\vec{r}.(2\hat{i} + 5\hat{j} + 3\hat{k}) = 0$

such that the intercepts made by the plane on x-axis and z-axis are equal.

24. In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3/5 be the probability that he knows the answer and 2/5 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1/3, what is the probability that the student knows the answer given that he answered it correctly?

25. A manufacturer produces nuts and bolts. It takes 2 hours work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A

and 2 hours on machine B to produce a package of bolts. He earns a profit of Rs. 24 per package on nuts and Rs. 18 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 10 hours a day. Make an L.P.P. from above and solve it graphically ?

26. The sum of surface areas of a sphere and a cuboid with sides $\frac{x}{3}$, x and $2x$, is constant. Show that the sum of their volumes is minimum if x is equal to three times the radius of sphere.